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WEIGHT OPTIMUM ARCH STRUCTURES

by

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Weight Optimum Arch Structures

by

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ABSTRACT

The goal of this investigation is to design minimum weight arch structures which span the distance between two points in two-dimensional space. An arch of unknown shape and variable cross-sectional width is modeled as a series of straight bar-beam elements. Finite Element Methods are used to compute the stresses in each element. Automated Design Synthesis (ADS) software is then used to vary the slope of each element and the cross-sectional width to prevent the yield stress of the material from being exceeded as ADS minimizes the arch volume to arrive at the minimum weight structure. Results are presented for a number of different loadings and boundary conditions.

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I. INTRODUCTION

A. MOTIVATION

Since the beginning of recorded history, the arch has been used in the design of a wide range of structures. The arch was initially developed as a more reliable and efficient way of building a portal in a wall comprised of bricks or stone blocks. Over the years, it has been applied to an ever increasing number of load bearing structures. A significant reason for its popularity with architects and engineers is the aesthetically pleasing form of the arch. Many designers still enjoy the idea of an ordinary person being moved to wonder how a structure manages to stand with so little visible support.

The development of the digital computer in the latter half of this century placed an incredibly powerful tool in the hands of designers who previously had to rely mainly on experience to select the best arch design for a given application. Using the high speed calculation capability of the computer, a designer could now use the techniques of numerical analysis to approximate the solution of a complex differential equation with a high degree of accuracy. These techniques could be used to model a structure under loading and obtain the optimum design. The arch has been the subject of several such studies.

In 1976, Farshad [ref. 1] derived two governing nonlinear partial differential equations for statically determinate, hinged-hinged arches. The system's total potential energy, modified by several objective functions through the use of Lagrange multipliers, was minimized in regards to design and state variables to achieve optimality and equilibrium. For an arch with specified span and loading, the nonlinear systems of equations for optimal thrust, minimum length, and minimum volume were presented, but not solved.

In 1980, Rozvany et al. [ref. 2] used the Prager-Shield criteria to determine the optimal shape of hinged-hinge frames. His frames consisted of two rigidly connected inclined beams with a point load applied at mid-span. He concluded that, for a single load condition, the optimal structure developed either bending only or axial forces only in the entire arch. When the ratio 4L/D, where L is the length of the span and D is the constant depth of the cross-section, is greater than eight, only axial forces develop. In this case, the optimal height of the arch is L/2. When 4L/D is less than eight, only bending develops and the optimal structure is a straight beam. The width of the cross-sections varied linearly from support to axis of symmetry. The author also found that although for a single system of point loads, the optimal arch consisted of straight segments, curved segments occurred if several alternative systems of point loads were considered.

Also in 1980, Lipson et al. [ref. 3] conducted a numerical study using an automated design routine to determine an optimal arch design with the arch shape and cross-sectional dimensions allowed to vary. He modelled a uniformly loaded parabolic arch as a system of straight segments with thin-walled, rectangular cross-sections. When he studied arches of constant depth and width, the wall thickness determined the optimal shape for minimum weight. The resulting shape was found to be a parabolic curve with a height of 0.342 times the span length.

In 1988, Ang et al. [ref.4] investigated the optimal shape of plastically designed non-funicular arches under a uniformly distributed load. The problem was solved by parameterizing the unspecified arch axis using spline functions and employing a smoothing function to approximate the non-smooth objective function (arch weight). The arch considered had a rectangular cross-section of variable width and constant depth. It carried a uniform load and its supports were either hinged-hinged, hinged-clamped, or clamped-clamped. The optimum shape of the arch was found to be a parabola with a height of 0.433 times the span length, which appears to disagree with Lipson's results.

In 1990, Charles Scott McDavid of the Naval Postgraduate School [ref. 5] investigated the optimization of circular arches subjected to various loading and boundary conditions. He modelled the arches as systems of straight segments with

constant depth and variable width. He concluded that the bar/beam model was a viable technique in the approximation of arch structures and that the more statically indeterminate an arch structure was, the more efficient it was under identical loading. Lieutenant McDavid also proposed several further topics of research in this areas. One of the recommended topics was taken up Margaret Anne Menzies in 1991 [ref. 6]. She investigated circular arches with varying depth and width. She also validated the bar/beam model as an approximation of an arch, but was limited by computer restrictions to a relatively small number of straight segments. This investigation looks into another area suggested by Lieutenant McDavid by allowing the radius of curvature (shape of centroidal axis) to be one of the design variables.

B. PROBLEM STATEMENT

The arch considered by this investigation has a rectangular cross-section with constant depth and varying thickness. The radius of curvature is allowed to vary to obtain the optimum shape of the arch. The distance spanned in the horizontal and vertical directions will be specified. The cross-sectional dimensions are assumed to be small with respect to the radius of curvature, which implies that the centroidal axis and the neutral axis coincide.

The governing equations for the behavior of the arch are the beam equilibrium equation:

$$(EIv'')'' = P_y(s) \tag{1}$$

and the bar equilibrium equation:

$$(AEu')'' = -P_x(s) \tag{2}$$

The prime superscript indicates a derivative with respect to the independent variable,s. The variables in the equations are defined as:

E = Young's Modulus

I = Cross-sectional Moment of Inertia

A = Cross-sectional Area

v = Lateral displacement

u = Axial displacement

P_y = Lateral loading

P_x = Axial loading

s = independent variable

The arch is approximated by a system of straight elements. The local displacements are used to generate the internal pseudo stresses by applying the virtual load techniques described by Ding and Esping [ref. 7]. When the stress distribution is known, the volume of the arch is minimized to obtain a shape that keeps the developed stresses below the maximum allowable stress.

The purpose of this study is to minimize the total weight of an arch spanning a specified horizontal and vertical distance under a variety of loading conditions. The material of the arch is assumed to be linearly elastic, homogeneous, and isotropic. The slope of the arch and the cross-sectional width at each node are allowed to vary to obtain the optimum (minimum weight) structure. Automated Design Synthesis software [ref. 8] is used to carry out the optimization.

II. PROBLEM FORMULATION

A. PROBLEM STATEMENT AND ASSUMPTIONS

As stated previously, the goal of this investigation is to design a minimum weight arch which spans a pre-defined distance in two-dimensional space. The radius of curvature and cross-sectional width of the arch are allowed to vary to obtain the optimum shape. The following assumptions are made to simplify the problem:

- The arch material is homogeneous, isotropic, and linearly elastic. This implies that the minimum volume arch is also the minimum weight arch.
- The arch is modelled as a series of straight bar-beam elements whose behavior is governed by equations 1 and 2.
- Failure is assumed to occur when the yield strength of the material is exceeded.

B. MATHEMATICAL MODEL

The arch spans a distance in two-dimensional space from point A to point B (see fig. 2.1). The number of elements, NEL, the cross-sectional height, h, and the horizontal distance spanned by each element, DX, are specified. DX is constant and is defined as:

$$DX = \Delta X = L/NEL \tag{3}$$

where L is the distance between A and B in the horizontal

direction.

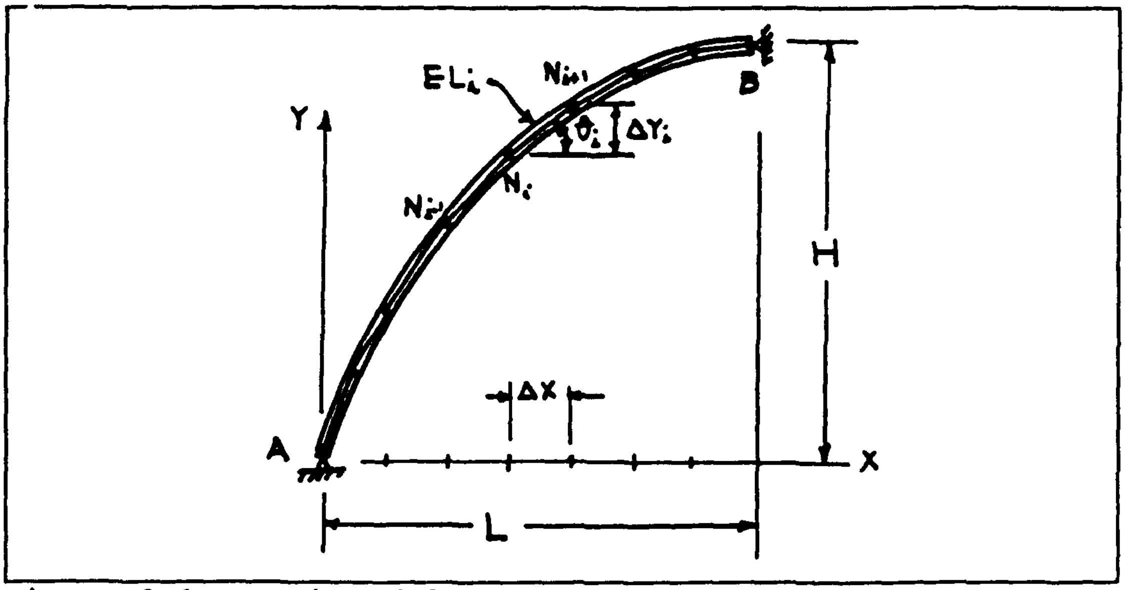

Figure 2.1: Arch Model

The design variables in this case are the cross-sectional base, b_i, at each node and the slopes, SLP_i, of the first NEL - 1 elements. SLP_i is defined as:

$$SLP_i = \theta_i = (Y_{i+1} - Y_i)/DX \tag{4}$$

where Y_{i+1} and Y_i are the vertical coordinates of the nodal points at the ends of the i^{th} element. Since the structure must rise a distance H, the slope of the last element is not a design variable, but is fixed by whatever values the slopes of the previous NEL-1 elements take.

The base dimension from one element to an adjacent element maintains smooth piece-wise continuity, as does a plot of the

spatial coordinates of the nodal points. The resulting elemental shape is a three-dimensional trapezoid (fig. 2.2) whose volume is determined by multiplying the average base and the height with the length of the element. The elemental length, l_i, is defined as:

$$l_i = \sqrt{DX^2 + (SLP_i * DX)^2} \tag{5}$$

Therefore, the volume of the i^{th} element is defined as:

$$\begin{aligned} &Volume_i = b_{ave_i} * h * l_i \\ &where: \\ &b_{ave_i} = (b_{i+1} + b_i)/2 \\ &b_i = nodal\ base\ dimension \\ &h = nodal\ height\ dimension \\ &l_i = elemental\ length \\ &i = i^{th}\ element \end{aligned} \tag{6}$$

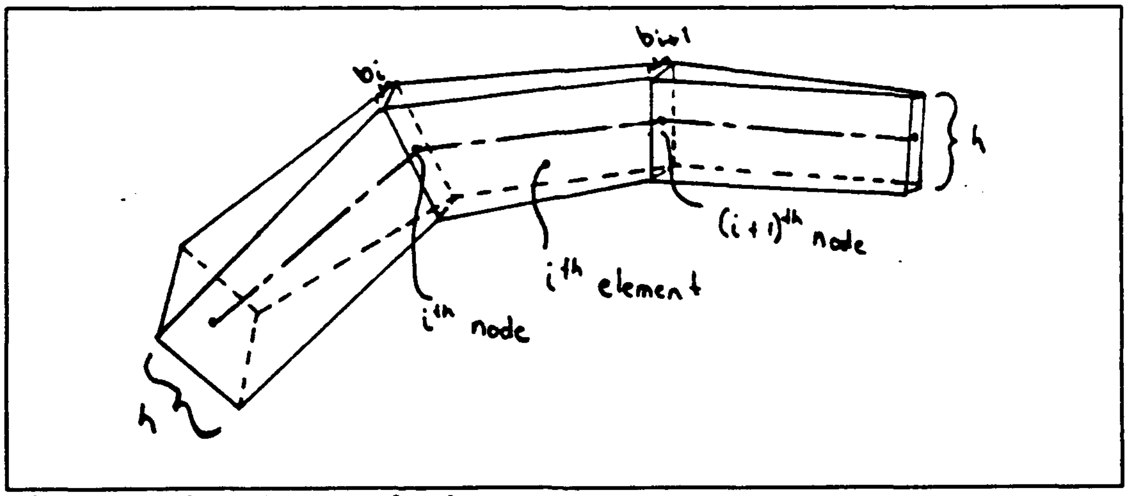

Figure 2.2: Elemental Shape

The volume of the entire arch is obtained by summing the individual elemental volumes.

C. OPTIMIZATION PROBLEM

The goal of this investigation is to obtain a minimum weight (volume) arch which maintains a stress distribution which does not exceed the yield strength of the material. The volume of the arch is defined as:

$$Volume = \sum_{I=1}^{NEL} Volume_I = \sum_{I=1}^{NEL} b_{ave_I} * h * l_I \qquad (7)$$

This equation can be rewritten as:

$$Volume = h * DX * (\sum_{I=1}^{NEL} b_{ave_I} * \sqrt{1 + SLP_I^2}) \qquad (8)$$

The objective function for the optimization problem is defined as:

$$OBJ = \sum_{I=1}^{NEL} b_{ave_I} * \sqrt{1 + (SLP_I)^2} \qquad (9)$$

Since h and DX are constant, minimizing the objective function will minimize the arch volume.

Constraints are imposed upon the objective function and design variables to ensure the applicability of the governing equations and to prevent failure by yielding. The strength constraint can be simply stated as follows:

$$\sigma_i \le S_y$$

or in normalized form:

$$\frac{\sigma_i}{S_y} - 1.0 \le 0.0 \qquad (10)$$

where:

σ_i = *maximum stress at* i^{th} *node*

S_y = *material yield strength*

The stress distribution in the arch is obtained by Finite Element Methods. This process is described in Chapter IV.

The constraints placed on the base dimensions of the arch are used to ensure the validity of the bar and beam equilibrium equations (equations (1) and (2)). Limiting the base relative to the constant height prevents the structure from behaving like a shell ar a deep curved beam. The following limits are imposed through the use of the ADS side constraints on b_i :

$$0.1*h \le b_i \le 3*h \qquad (11)$$

It is also important to choose a cross- sectional height dimension that is small relative to the length of the entire structure.

One limit on the cross-sectional dimensions that was not considered by this investigation is that imposed by the requirement for elastic stability. The obvious importance of avoiding failure by buckling is an interesting area for future research.

III. OPTIMIZATION ANALYSIS

The computer optimization is performed by ADS, a general purpose numerical optimization program containing a wide variety of algorithms. The program minimizes one function, the objective, subject to bounds imposed on the design variables by constraint functions. The solution of the general problem is separated into three basic levels which are strategy, optimizer, and one-dimensional search. For this constrained minimization problem, the ADS user's guide [ref. 8] recommends a Sequential Linear Programming strategy, a Modified Method of Feasible Directions optimizer, and a Golden Section Method one-dimensional search.

A. STRATEGY

Sequential Linear Programming (SLP), linearizes nonlinear objective and constraint functions, then obtains a solution to the approximation of the problem by linear programming methods (see fig.3.1). The problem is linearized again about the design point yielded by the solution to the previous approximation and resolved. This process is repeated until a precise solution is obtained [ref. 9].

The nonlinear functions are linearized through the use of a first-order Taylor Series expansion in the following manner:

$$
\begin{aligned}
&\textit{Minimize:} \quad F(\vec{X}) = F(\vec{X}_o) + \nabla F(\vec{X}_o) * \delta\vec{X} \\
&\textit{Subject to:} \quad g_j(\vec{X}_o) + \nabla g_j(\vec{X}_o) * \delta\vec{X} \le 0 \\
&\qquad \textit{where:} \quad \delta\vec{X} = \vec{X} - \vec{X}_o \\
&\qquad j = j^{th} \textit{ constraint}
\end{aligned}
\tag{12}
$$

The zero subscript identifies the point about which the Taylor series expansion is performed. At the initial design, the objective and constraints are linearized to give straight line representations of the functions.

In an under-constrained problem like this one, where there are fewer active constraints than design variables, this method sometimes performs poorly. This occurs because the linear approximation may be unbounded. This problem is dealt with by imposing move limits on the linear approximation as shown in figure 3.1. This ensures that the optimum is eventually reached within a tolerance of the move limits. In practice, these move limits are reduced during the design process so that a solution is found with the desired accuracy. SLP tends to converge rapidly to a solution, but while the solution of the linear problem is near the nonlinear optimum, it is in the infeasible region.

B. OPTIMIZER

The Modified Method of Feasible Directions for constrained minimization is used to solve the approximate optimization

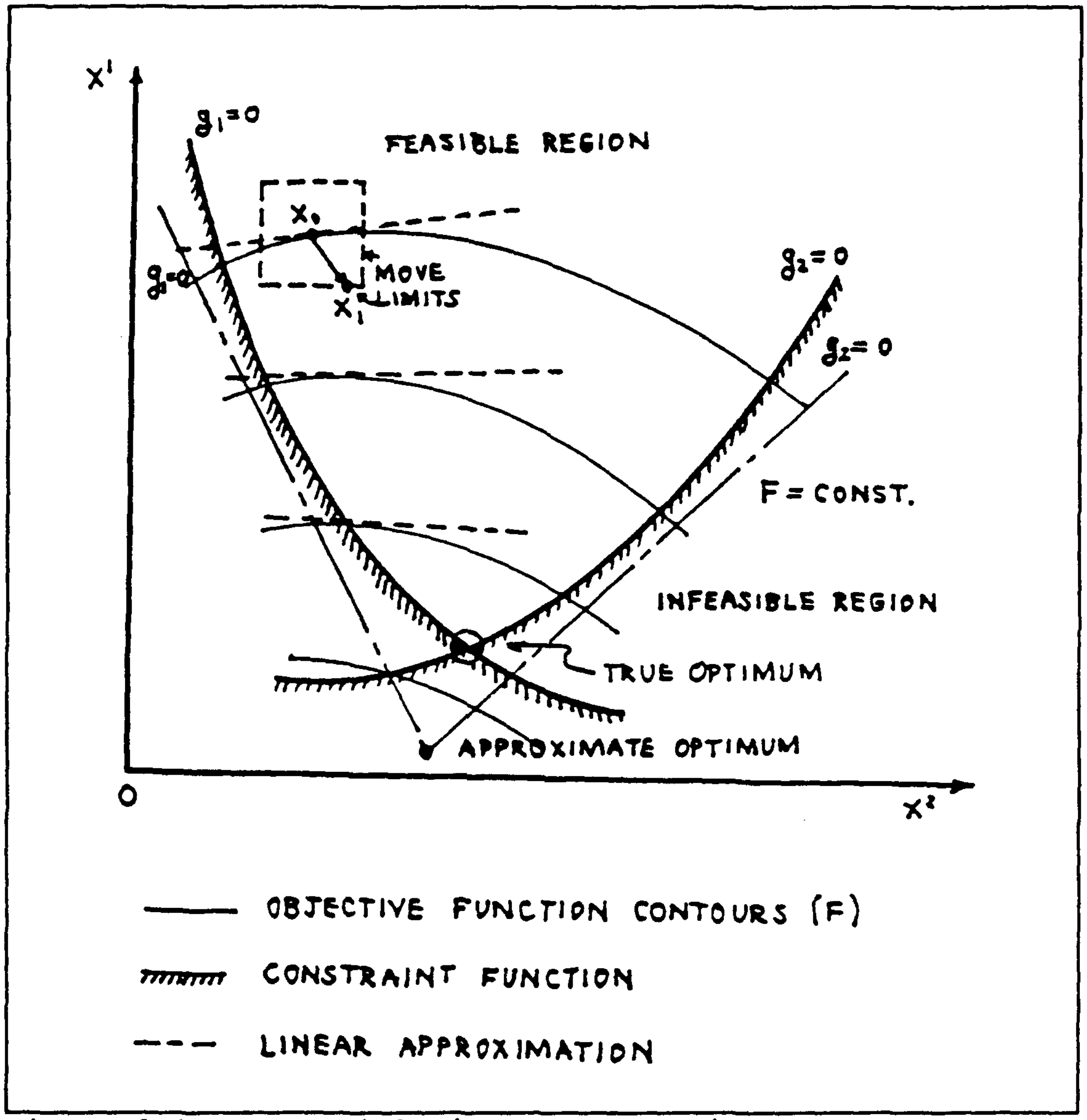

Figure 3.1: Sequential Linear Programming

sub-problem (eq. 12). In this method, a search direction vector is first found. The vector containing the design variables is then updated by moving in the direction of the search vector in the following manner (refer to fig. 3.2):

$$\vec{X}_q = \vec{X}_{q-1} + \alpha^* \vec{S}_q \tag{13}$$

The scalar α^* defines the distance moved in the direction of the search vector in design variable space and q represents the iteration number. From the initial design, the objective function is moved in the direction of steepest descent in design variable space. This will continue until a constraint is encountered. Once this happens, a new search direction is obtained by solving the sub-problem:

$$\begin{aligned} &\textit{Maximize}: \quad -\nabla F(\vec{X}) * \vec{S} \\ &\textit{Subject to}: \quad \nabla g_j(\vec{X}) * \vec{S} \le 0 \qquad j \in J \\ &\qquad\qquad \vec{S} * \vec{S} \le 1 \end{aligned} \tag{14}$$

where:

$\vec{X}$ = *vector whose elements are the design variables*

F = *the objective function*

$\vec{S}$ = *the search vector*

J = *number of active constraints*

The search direction will follow the constraint, but will allow the design to leave the constraint boundary if the objective function can be further reduced. If the scalar product of the gradient of each critical constraint with the search vector is less than zero, the search vector is moving away from the currently active constraint. This constraint is then dropped from the set of active constraints. If the search vector is the null vector, or numerically small, the optimization is terminated because this indicates that the Kuhn-Tucker conditions for optimality have been met [ref. 9].

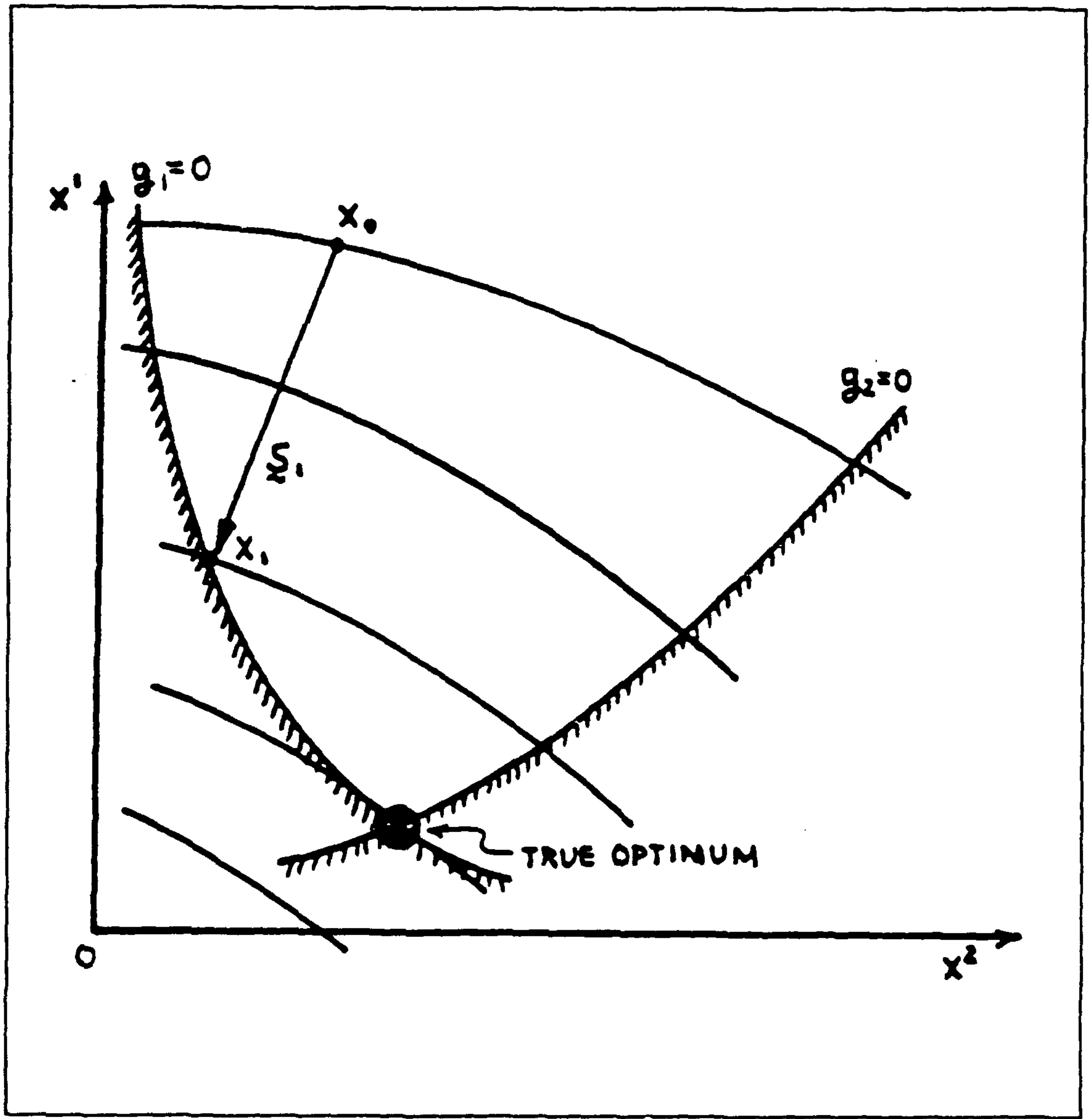

Figure 3.2: Modified Method of Feasible Directions

C. ONE-DIMENSIONAL SEARCH

The scalar α^* in equation (13) is found by the Golden Section Method. Basically, this determines how far the optimizer will search in the direction of the search vector. The one-dimensional search attempts to find the optimal value

of α^* which will result in the minimum value of the objective function. It accomplishes this by progressively bracketing the minimum of the function by comparing the values of the function at smaller and smaller limits.

D. ADS PROGRAM PARAMETERS

Reference 8 contains a table of ADS program parameters. It is possible to change these parameters from their default values with the first call to ADS using an INFO value equal to -2. The purpose of modifying these parameters is to try to obtain a better optimal design by "fine tuning" the program. For instance, modifying the constraint tolerance, CT, can ensure that the active constraints will be closer to the desired value. However, this can also mean that it will take much longer for the solution to converge. The only parameter modification done in this investigation was to inhibit the auto-scaling function of ADS.

IV. STRESS ANALYSIS

In order to accomplish the goal of designing a minimum weight arch, a FORTRAN computer was used to evaluate the objective function and constraints and to invoke the ADS software. The strength constraints require that the stress at each nodal point remain below the yield strength of the material. Determining the stress distribution in the arch is a complex problem, because of the need to be able to analyze a statically indeterminate structure. Finite Element Methods are used to calculate the displacements at each nodal point. The displacements are then related to the nodal forces and moments by the direct stiffness method. Once the forces and moments are known, the stresses can be evaluated with a knowledge of the cross-sectional dimensions.

A. STRESS DEVELOPMENT

For the purposes of this investigation, the arch is assumed to fail if the total stress at any nodal point exceeds the yield stress of the material. The total stress is defined as the sum of the normal stresses due to bending and axial loading.

$$\sigma_t = |\sigma_b| + |\sigma_a|$$

where:

σ_t = total normal stress (15)

σ_b = normal stress due to bending

σ_a = normal stress due to axial load

The absolute values of the bending and axial stresses are used because at either the top or bottom extreme fiber the stresses will be additive. In other words, the yield stress is assumed to be the same in compression and tension. There will be shear stresses developed in the arch, but the geometric limits imposed by the side constraints ensure that these stresses remain negligible (see Appendix A).

To calculate the two normal stress components, the arch is modelled as a series of straight elements, which can be considered to behave as beams to calculate the bending stresses and as bars to calculate the axial stress. The bending stress in an element is defined as:

$$\sigma_b = \frac{Mc}{I}$$

where:

M = bending moment (16)

c = distance from centroidal axis to extreme fiber

I = Moment of Inertia of cross-section

The moment, M, is calculated by:

$$M = EIv''$$

where:

E = Young's Modulus (17)

v = lateral displacement

By substituting equation (17) into equation (16) the following equation is obtained:

$$\sigma_b = Ecv'' \qquad (18)$$

In a similar manner, the axial stress is calculated. The stress due to axial loading is defined as:

$$\sigma_a = \frac{F}{A}$$

where: (19)

F = axial load

A = cross-sectional area

The axial force, F, is determined by the following equation:

$$F = AEu'$$

where: (20)

u = axial displacement

By substituting equation (20) into equation (19), the following result is achieved:

$$\sigma_a = Eu' \qquad (21)$$

Combining equations (18) and equation (21) results in:

$$\sigma_t = E(cv'' + u') \qquad (22)$$

The approximate values of the lateral and axial displacements are obtained through the use of the Galerkin Finite Element Method. With the use of these values, the stresses at each nodal point can be calculated.

B. FINITE ELEMENT BEAM EQUATION DEVELOPMENT

The Galerkin Finite Element Method is used to transform the fourth-order differential equation governing static displacement of the beam into a system of linear algebraic equations. In order to maintain continuity of slopes and displacements from element to element, a family of cubic shape functions is introduced [ref. 5 and ref. 6]. An approximate solution for lateral displacement is defined as:

$$v \approx \hat{v} = \vec{N}^T \vec{v}$$

where:

v = *exact solution in continuous space* **(23)**

$\hat{v}$ = *approximate solution in discrete space*

$\vec{N}$ = *vector containing shape functions*

$\vec{v}$ = *vector containing lateral displacements and slopes*

The next step is to form a measure of the error of the approximation, or residual:

$$R = [EI\hat{v}''] - p_y(s) \qquad \textbf{(24)}$$

where p_y is the lateral load and s is the independent variable. Substituting equation (23) into equation (24) yields the following equation:

$$R = [EI(\vec{N}^T\vec{v})'']'' - p_y(s) \tag{25}$$

The residual can be minimized if:

$$\int_D \vec{N}R(s)\,ds = \vec{0} \tag{26}$$

where the quantity on the right hand side of the equation is the null vector. Substitution of equation (25) into equation (26) gives the following result:

$$\int_D \vec{N}[EI(\vec{N}^T\vec{v})'']''ds - \int_D \vec{N}p_y(s)\,ds = \vec{0} \tag{27}$$

Integration by parts is performed twice on equation (27), resulting in:

$$\begin{aligned} &\vec{N}[EI(\vec{N}^T\vec{v})'']'|_B - (\vec{N})'EI(\vec{N}^T\vec{v})''|_B \\ &+ \int_D (\vec{N})''EI(\vec{N}^T\vec{v})''ds - \int_D \vec{N}p_y(s)\,ds = \vec{0} \end{aligned} \tag{28}$$

where $|_B$ indicates an evaluation at the boundary points of the structure. Since the displacement vector is a constant, equation (28) can be rewritten as:

$$\begin{aligned} &\vec{N}[EI(\vec{N}^T)'']'\vec{v}|_B - (\vec{N})'EI(\vec{N}^T)''\vec{v}|_B \\ &+ \int_D (\vec{N})''EI(\vec{N}^T)''ds\vec{v} - \int_D \vec{N}p_y(s)\,ds = \vec{0} \end{aligned} \tag{29}$$

From the beam equilibrium equation, the shear force, V, is defined as:

$$V = EIv''' \tag{30}$$

and the moment, M, by:

$$M = EIv'' \tag{31}$$

Therefore, the boundary term load vectors are defined as:

$$\vec{V} = \vec{N}[EI(\vec{N}^T)'']'\vec{v}|_B \tag{32}$$

and:

$$\vec{M} = (\vec{N})'EI(\vec{N}^T)''\vec{v}|_B \tag{33}$$

A system stiffness matrix is defined as:

$$\overline{K}_B = \int_D (\vec{N})''EI(\vec{N}^T)''ds \tag{34}$$

and a system force vector by:

$$\vec{F}_b = \int_D \vec{N}p_y(s)\,ds \tag{35}$$

Substitution of equations (32) through (35) into equation (29) yields the following system of linear equations:

$$\vec{V}|_B - \vec{M}|_B + \overline{K}_B\vec{v} - \vec{F}_b = \vec{0} \tag{36}$$

If a load vector of internal and external applied loads is defined as:

$$\vec{F}_B = \vec{F}_b + \vec{M}|_B - \vec{V}|_B \tag{37}$$

equation (36) reduces to:

$$\overline{K}_B\vec{v} = \vec{F}_B \tag{38}$$

The global bending stiffness matrix is constructed from the union of the elemental bending stiffness matrices and the global bending force vector is constructed from the union of the elemental bending force vectors.

C. FINITE ELEMENT BAR EQUATION DEVELOPMENT

The development of the Galerkin Finite Element Method for the bar equilibrium equation is done in a manner similar to that for the beam equation. However, since the bar equation is a first-order differential equation, a family of linear shape functions can be used to maintain continuity of displacement from element to element [ref. 5 and ref. 6]. Once again, an approximate solution for the axial displacement, u, is formed as follows:

$$u \approx \tilde{U} = \vec{N}^T\vec{U}$$

where:

u = *exact solution in continuous space*

$\tilde{U}$ = *approximate solution in discrete space* (39)

$\vec{N}$ = *vector containing shape functions*

$\vec{U}$ = *vector containing axial displacements*

The residual is defined as:

$$R(s) = [AE(\vec{N}^T\vec{U})']' + p_x(s) \tag{40}$$

The residual can be minimized if:

$$\int_D \vec{N}[R(s)]\,ds = \vec{0} \tag{41}$$

Substituting equation (40) into equation (41) results in the following equation:

$$\int_D \vec{N}[AE(\vec{N}^T\vec{U})']'ds + \int_D \vec{N}p_x(s)\,ds = \vec{0} \tag{42}$$

Unlike the beam equation development, only one integration by parts is performed to yield:

$$AE\vec{N}(\vec{N}^T\vec{u})'|_B - \int_D (\vec{N})'[AE(\vec{N}^T\vec{u})']\,ds + \int_D \vec{N}p_x(s)\,ds = \vec{0} \qquad (43)$$

where $|_B$ indicates an evaluation at the boundary points of the structure. Since the axial displacement vector is a constant, equation (43) can be rewritten as:

$$\vec{N}(AE\vec{N}^T)'\vec{u}|_B - \int_D (\vec{N})'[AE(\vec{N}^T)']\,ds\vec{u} + \int_D \vec{N}p_x ds = \vec{0} \qquad (44)$$

From the bar equilibrium equation, the axial force, F, is defined as:

$$F = AEu' \qquad (45)$$

Therefore, the boundary term load vector vectors are defined as:

$$\vec{p} = AE\vec{N}(\vec{N}^T)'\vec{u}|_B \qquad (46)$$

A stiffness matrix is defined as:

$$\overline{K}_A = \int_D (\vec{N})'[AE(\vec{N}^T)]\,ds \qquad (47)$$

and a system force vector by:

$$\vec{F}_s = \int_D \vec{N}p_x(s)\,ds \qquad (48)$$

Substituting equations (46) through (48) into equation (44) yields the following equation:

$$\vec{p} - \overline{K}_A\vec{u} + \vec{F}_s = \vec{0} \qquad (49)$$

By defining a new load vector as:

$$\vec{F}_A = \vec{F}_B + \vec{p} \tag{50}$$

equation (49) reduces to:

$$\overline{K}_A \vec{u} = \vec{F}_A \tag{51}$$

The global axial stiffness matrix is constructed from the union of the elemental axial stiffness matrices and the global axial force vector is constructed from the union of the elemental axial force vectors.

D. THE ELEMENTAL STIFFNESS MATRIX

The global Galerkin FEM stiffness matrices in equations (38) and (51) are constructed from the union of elemental axial and bending stiffness matrices. A single beam element has four degrees of freedom, which yields a 4x4 elemental stiffness matrix. A single bar element has two degrees of freedom, which yields a 2x2 elemental stiffness matrix. These elements can be combined into a single bar-beam element (fig. 4.1) which has six degrees of freedom [ref. 5 and ref. 6].

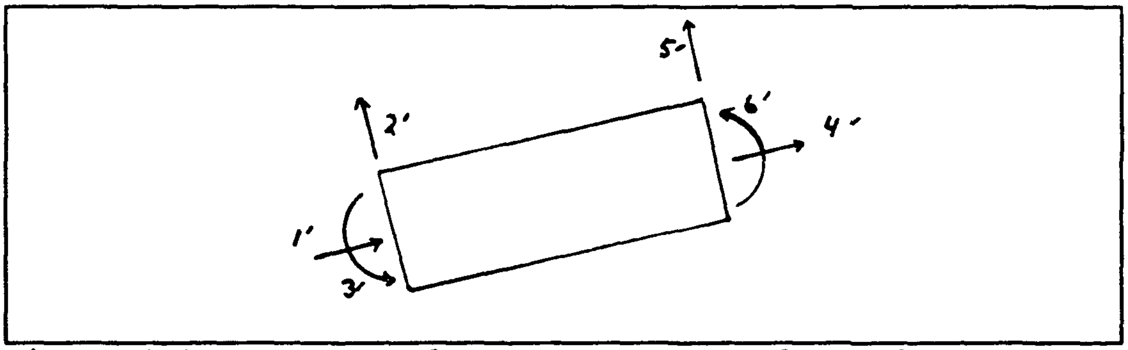

Figure 4.1: Bar-Beam Element - Degrees of Freedom

This results in a 6x6 elemental stiffness matrix. The elemental displacement vector can be expressed as:

$$(\bar{\delta}^{i'})^T = \langle \delta_1^{i'}, \delta_2^{i'}, \delta_3^{i'}, \delta_4^{i'}, \delta_5^{i'}, \delta_6^{i'} \rangle$$

where for the i^{th} *element*:

$\delta_1^{i'}$ = *axial displacement at local node* 1

$\delta_2^{i'}$ = *lateral displacement at local node* 1

$\delta_3^{i'}$ = *beam slope at local node* 1 (52)

$\delta_4^{i'}$ = *axial displacement at local node* 2

$\delta_5^{i'}$ = *lateral displacement at local node* 2

$\delta_6^{i'}$ = *beam slope at local node* 2

The elemental force vector can be expressed as:

$$(\bar{f}^{i'})^T = \langle f_1^{i'}, f_2^{i'}, f_3^{i'}, f_4^{i'}, f_5^{i'}, f_6^{i'} \rangle$$

where for the i^{th} *element*:

$f_1^{i'}$ = *axial force at local node* 1

$f_2^{i'}$ = *lateral force at local node* 1

$f_3^{i'}$ = *moment at local node* 1 (53)

$f_4^{i'}$ = *axial force at local node* 2

$f_5^{i'}$ = *lateral force at local node* 2

$f_6^{i'}$ = *moment at local node* 2

Therefore, the combination of the Galerkin bar and beam equations yields:

$$\bar{k}^{i'}\bar{\delta}^{i'} = \bar{f}^{i'} \tag{54}$$

where the elemental stiffness matrix takes the following form:

$$\overline{k}^{i'} = \begin{bmatrix} \frac{AE}{l_i} & 0 & 0 & -\frac{AE}{l_i} & 0 & 0 \\ 0 & 12\frac{EI}{l_i^3} & 6\frac{EI}{l_i^2} & 0 & -12\frac{EI}{l_i^3} & 6\frac{EI}{l_i^2} \\ 0 & 6\frac{EI}{l_i^2} & 4\frac{EI}{l_i} & 0 & -6\frac{EI}{l_i^2} & 2\frac{EI}{l_i} \\ -\frac{AE}{l_i} & 0 & 0 & \frac{AE}{l_i} & 0 & 0 \\ 0 & -12\frac{EI}{l_i^3} & -6\frac{EI}{l_i^2} & 0 & 12\frac{EI}{l_i^3} & -6\frac{EI}{l_i^2} \\ 0 & 6\frac{EI}{l_i^2} & 2\frac{EI}{l_i} & 0 & -6\frac{EI}{l_i^2} & 4\frac{EI}{l_i} \end{bmatrix} \quad (55)$$

It is apparent from the form of the elemental stiffness matrix that the bar and beam have uncoupled behavior.

E. COORDINATE TRANSFORMATION OF THE ELEMENTAL SYSTEM OF EQUATIONS

Each element in the arch will have a unique orientation with respect to the global x and y axes. In order to solve the global system of equations, it is necessary to transform each elemental Galerkin equation into global coordinates. The global reference coordinate system is defined as the horizontal and vertical axes of the entire arch. Each element makes an angle α_i with the horizontal axis and an angle β_i with the vertical axis.

The local displacements and forces (indicated by prime superscript) are defined as follows:

$$
\begin{aligned}
\delta_1^{i'} &= \delta_1^{i}\cos(\alpha_i) + \delta_2^{i}\cos(\beta_i) \\
\delta_2^{i'} &= -\delta_1^{i}\cos(\beta_i) + \delta_2^{i}\cos(\alpha_i) \\
\delta_3^{i'} &= \delta_3^{i} \\
\delta_4^{i'} &= \delta_4^{i}\cos(\alpha_i) + \delta_5^{i}\cos(\beta_i) \\
\delta_5^{i'} &= -\delta_4^{i}\cos(\beta_i) + \delta_5^{i}\cos(\alpha_i) \\
\delta_6^{i'} &= \delta_6^{i}
\end{aligned}
\tag{56}
$$

and:

$$
\begin{aligned}
f_1^{i'} &= f_1^{i}\cos(\alpha_i) + f_2^{i}\cos(\beta_i) \\
f_2^{i'} &= -f_1^{i}\cos(\beta_i) + f_2^{i}\cos(\alpha_i) \\
f_3^{i'} &= f_3^{i} \\
f_4^{i'} &= f_4^{i}\cos(\alpha_i) + f_5^{i}\cos(\beta_i) \\
f_5^{i'} &= -f_4^{i}\cos(\beta_i) + f_5^{i}\cos(\alpha_i) \\
f_6^{i'} &= f_6^{i}
\end{aligned}
\tag{57}
$$

Therefore a transformation matrix can be defined as:

$$
\Gamma^{i} = \begin{bmatrix}
\cos(\alpha_i) & \cos(\beta_i) & 0 & 0 & 0 & 0 \\
-\cos(\beta_i) & \cos(\alpha_i) & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos(\alpha_i) & \cos(\beta_i) & 0 \\
0 & 0 & 0 & -\cos(\beta_i) & \cos(\alpha_i) & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{58}
$$

This reduces equations (56) and (57) to:

$$
\boldsymbol{\delta}^{i'} = \boldsymbol{\Gamma}^{i}\boldsymbol{\delta}^{i}
\tag{59}
$$

and:

$$\bar{f} = \bar{\Gamma}^i \bar{\delta}^i \tag{60}$$

where:

$$(\bar{\delta}^i)^T = \langle \delta_1{}^i, \delta_2{}^i, \delta_3{}^i, \delta_4{}^i, \delta_5{}^i, \delta_6{}^i \rangle \tag{61}$$

$$(\bar{f}^i)^T = \langle f_1{}^i, f_2{}^i, f_3{}^i, f_4{}^i, f_5{}^i, f_6{}^i \rangle \tag{62}$$

The transformed elemental stiffness equation becomes:

$$\bar{k}^{i\prime}\, \bar{\Gamma}^i \bar{\delta}^i = \bar{\Gamma}^i \bar{f}^i \tag{63}$$

Multiplying both sides of equation (63) by the inverse of the transformation matrix, which is an orthogonal matrix, yields:

$$(\bar{\Gamma}^i)^T \bar{k}^{i\prime} (\bar{\Gamma}^i) \bar{\delta}^i = \bar{f}^i \tag{64}$$

where the elemental stiffness matrix in terms of global coordinates is defined as:

$$\bar{k}^i = (\bar{\Gamma}^i)^T \bar{k}^{i\prime} (\bar{\Gamma}^i) \tag{65}$$

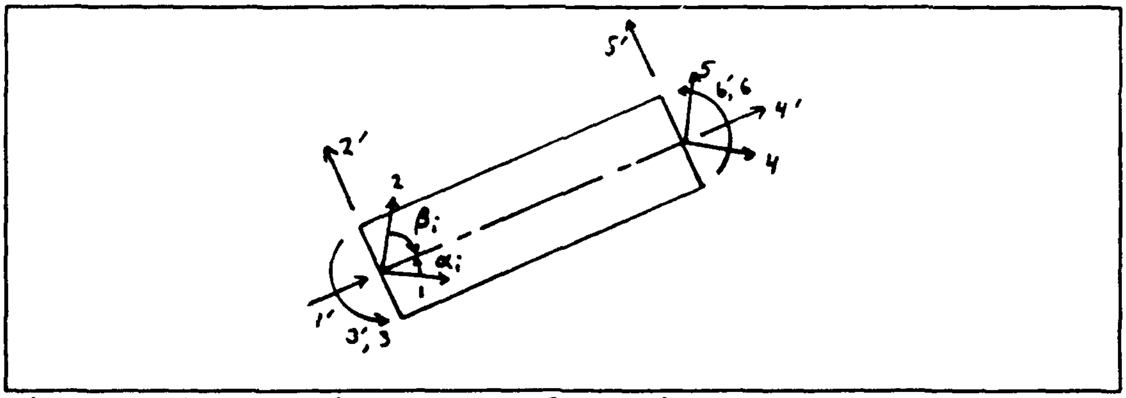

Figure 4.2: Coordinate Transformation

F. SOLUTION

The global stiffness matrix and global force vector are constructed from the union of the elemental stiffness matrices and elemental force vectors, which yields the following equation:

$$
\begin{aligned}
&\overline{K}\,\vec{\Delta} = \vec{F} \\
&\textit{where:} \\
&\overline{K} = \textit{global stiffness matrix} \\
&\vec{\Delta} = \textit{global displacement vector} \\
&\vec{F} = \textit{global force vector}
\end{aligned}
\tag{66}
$$

By multiplying both sides of the equation by the inverse of the global stiffness matrix, the global displacement vector can be calculated. The global displacements are then transformed back into the local displacements and can be used in conjunction with the elemental stiffness matrix to calculate the local forces:

$$\overline{k}^{i'}\vec{\delta}^{i'} = \vec{f}^{i'} \tag{67}$$

These local forces and bending moments are used to calculate the stress at each nodal point. The stresses of internal global nodal points are averaged since local nodal point 2 of the i^{th} element is the same as local nodal point 1 of the $(i+1)^{th}$ element.

V. PROGRAM DESCRIPTION

The Fortran 77 code originally used in references 5 and 6 to analyze circular arches was modified to analyze the non-circular arches of interest in this study. A copy of the program is included in Appendix B. The program reads information from an input file which provides the following user-supplied data:

L = horizontal distance spanned by arch

H = vertical distance spanned by arch

HGT = depth of cross-section

YOUNG = Young's Modulus

YIELD = yield strength

NEL = number of elements

ISTRAT = ADS parameter which designates strategy

IOPT = ADS parameter which designates optimizer

IONED = ADS parameter which designates one-d search method

IPRINT = ADS print control parameter

IGRAD = ADS parameter which designates method of gradient calculation

DV_BG = initial value of design variable

DV_LO = lower side constraint on design variable

DV_UP = upper side constraint on design variable

CLAN = node at which concentrated load is applied

FX = concentrated horizontal force (positive to the right) in pounds

FY = concentrated vertical force (positive upwards) in pounds

FM = concentrated moment (positive counter-clockwise) in pound-inches

FA = distributed load in pounds per inch

BX_ = boundary condition on horizontal displacement at 1, first node, or 2, last node (= 1 - fixed,= 0 - free)

BY_ = boundary condition on vertical displacement at 1, first node, or 2, last node (= 1 - fixed,= 0 - free)

BM_ = boundary condition on rotation at 1, first node, or 2, last node (= 1, - fixed,= 0 - free)

LABEL = string character identifying particular case study

The program then utilizes several subroutines to perform the FEM analysis and call the ADS program to perform the optimization.

An outline of the program, ARCHOPT, and its subroutines is provided in fig. 5.1. The first subroutine called by the main program is OPTIMIZATION_TOOL, which establishes the ADS parameters before the first call of the ADS program. The first call serves to override some of the default parameters in order to "fine-tune" the program. After ADS is called, OPTIMIZATION_TOOL calls subroutine EVAL to evaluate the objective function and constraints, which are functions of the design variables.

Subroutine EVAL calls subroutine ARCH_STRESS, which in turn calls subroutines FORM and FORCE_VECTOR. The latter subroutines are used to form the global stiffness matrix and

the global force vector, respectively. Subroutine BNDRY imposes the user-supplied boundary conditions by modifying the stiffness matrix and the force vector. An equation solver from the IMSL library, L2ARG, is used to solve the Galerkin FEM equation by matrix inversion. Once the global displacements are known,they can be used to calculate the nodal stresses, which is done in subroutine STRESS. Once the design constraints are evaluated for the initial design, the problem is returned to ADS, where an updated design is chosen. This process continues until the solution converges.

Once the termination criteria for optimization is satisfied, the main program calls subroutine ARCH_OUT, which creates an output file, ARCHOUT, which contains the problem parameters, the optimized design, and the arch volume. Copies of the output files for all case studies are provided in Appendix D.

The methodology of the program was thoroughly validated in references 5 and 6. Once the modifications were made, the program was used to analyze several statically determinate structures (i.e. a cantilever beam) to verify that the approximate values of stress and displacement came close to the exact values. The results confirmed that the FEM analysis provided an excellent approximation, with less than 0.8% error for a twelve element model of a cantilever beam with a lateral concentrated load at the free end.

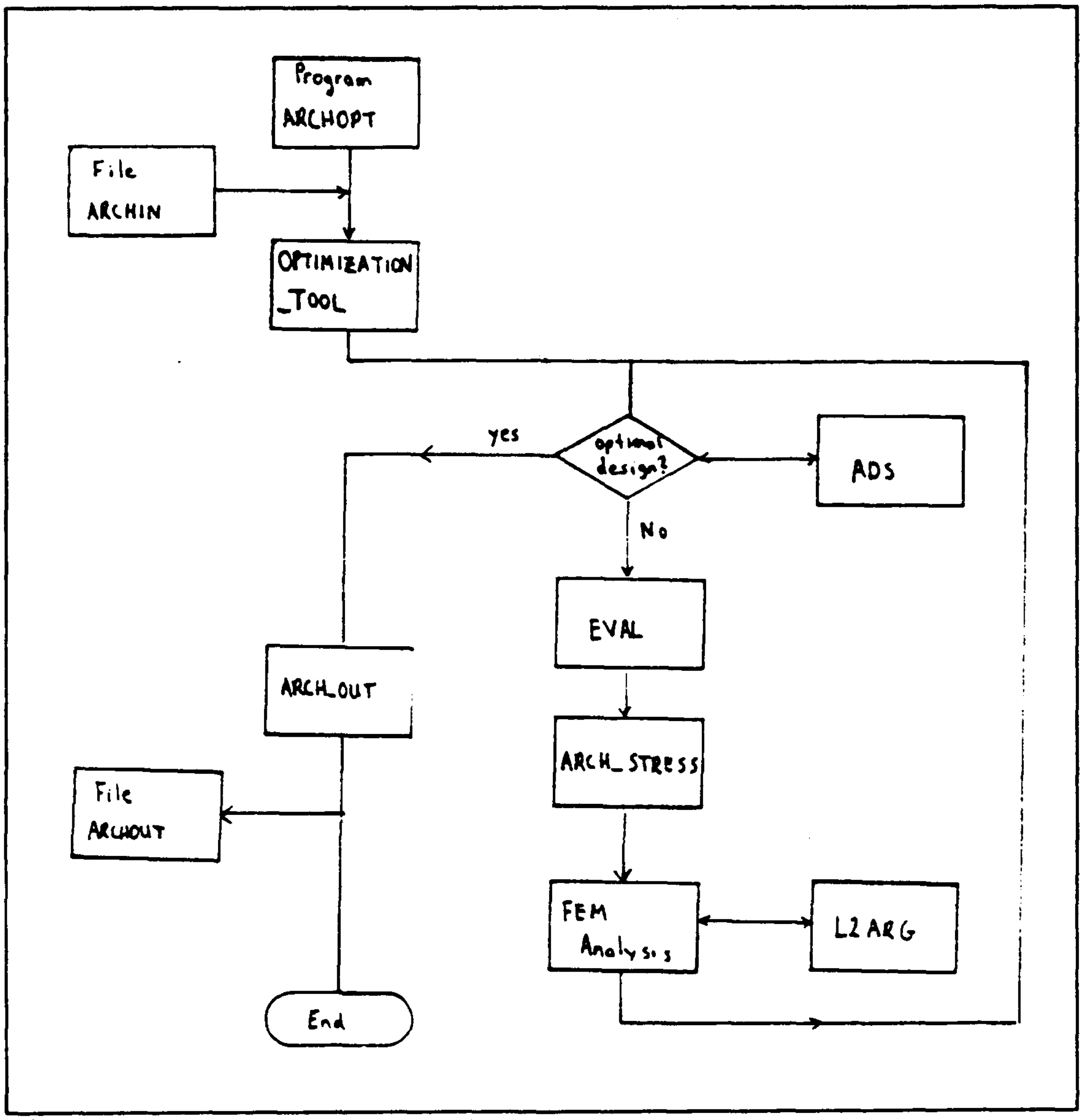

Figure 5.1: Program ARCHOPT Structure

VI. CASE STUDIES

The optimized designs for eleven different cases of loading and boundary conditions are presented. Cases (1) through (4) are hinged-hinged arches which span equal distances in the horizontal and vertical directions. Each case has a different loading. Cases (5) through (8) are arches which span equal horizontal and vertical distances, but which have different boundary conditions. Cases (9) and (10) are hinged-hinged arches which span different distances in the vertical direction. Plots of arch shape, base dimensions, and stress are provided for each case. It should be note that the distributed loads are applied in a direction normal to each element. This would closely approximate a situation where the arch was under hydrostatic, or pressure loading. Twelve elements were used to model the arch in all cases. This implies that there are thirteen nodal points and thirteen stress constraints. A Young's Modulus of 30,000,000 psi and a S_y of 52,000 psi are used in all cases. The output files from program ARCHOPT are presented in Appendix C.

A. CASE 1: HINGED-HINGED ARCH WITH DISTRIBUTED LOAD

This design has nine active stress constraints and thirteen active side constraints (lower limit on base). The stress due to axial loading is higher than the bending stress at all nodes. This indicates that the minimum area cross-section is better able to withstand an axial load than a bending moment. A predominantly axial stress distribution is more efficient since the entire cross-section is stressed to the same degree. In the case of a bending stress distribution, the material has to be strong enough to withstand the extreme fiber stress, which means that the internal part of the cross-section will be stressed below the yield strength.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed load = -100 pounds/inch
- Volume = 10.324 cubic inches

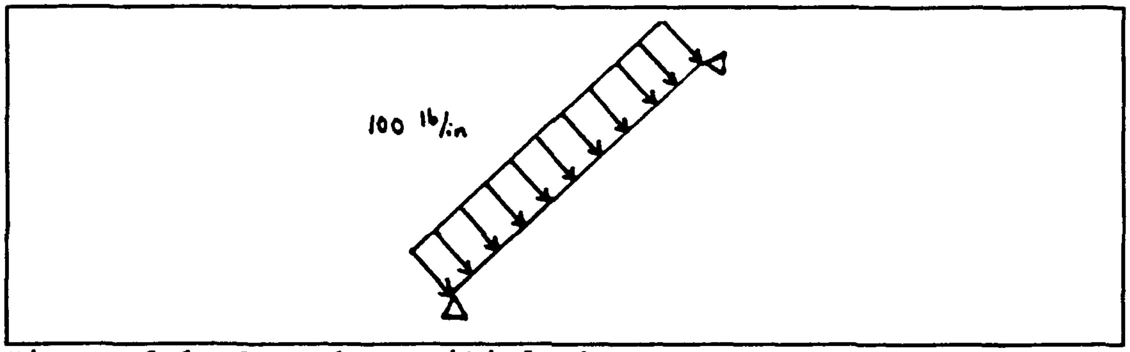

Figure 6.1: Case 1 - Initial Shape

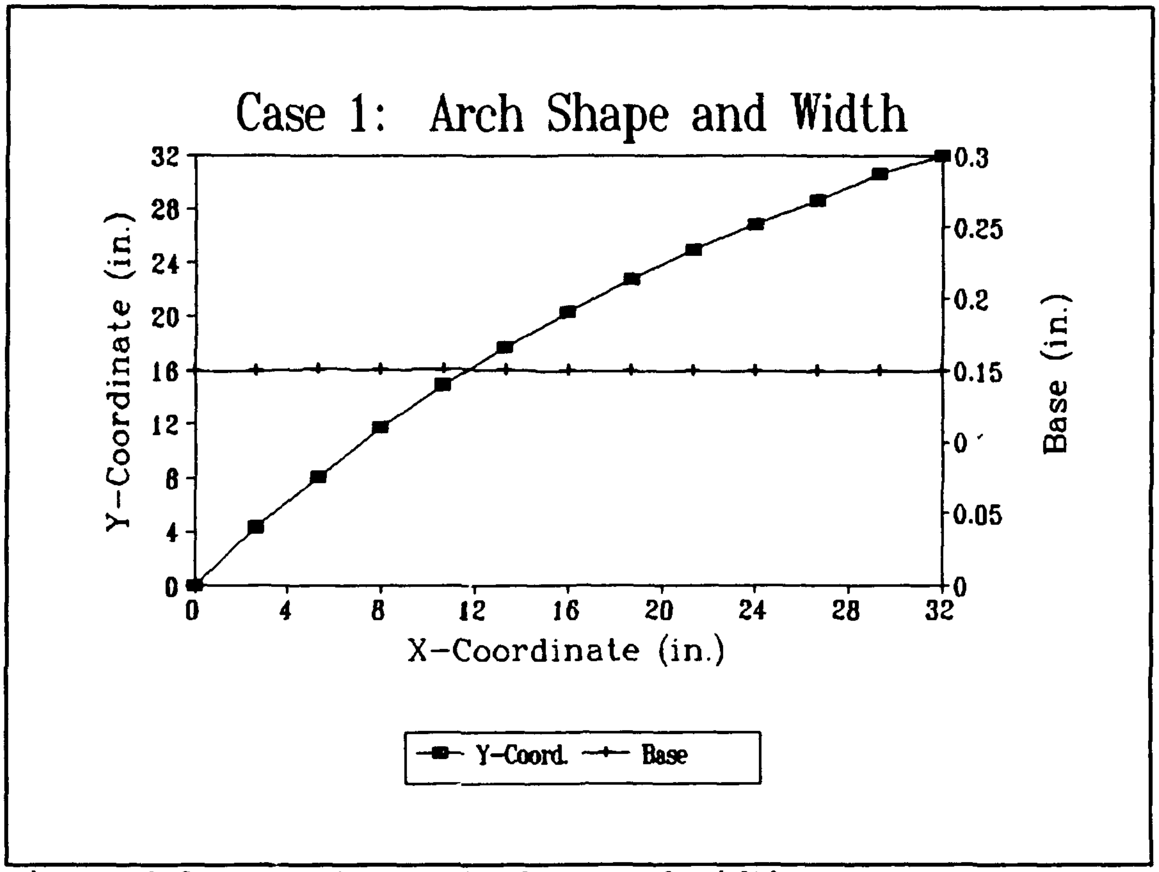

Figure 6.2: Case 1 - Arch Shape and Width

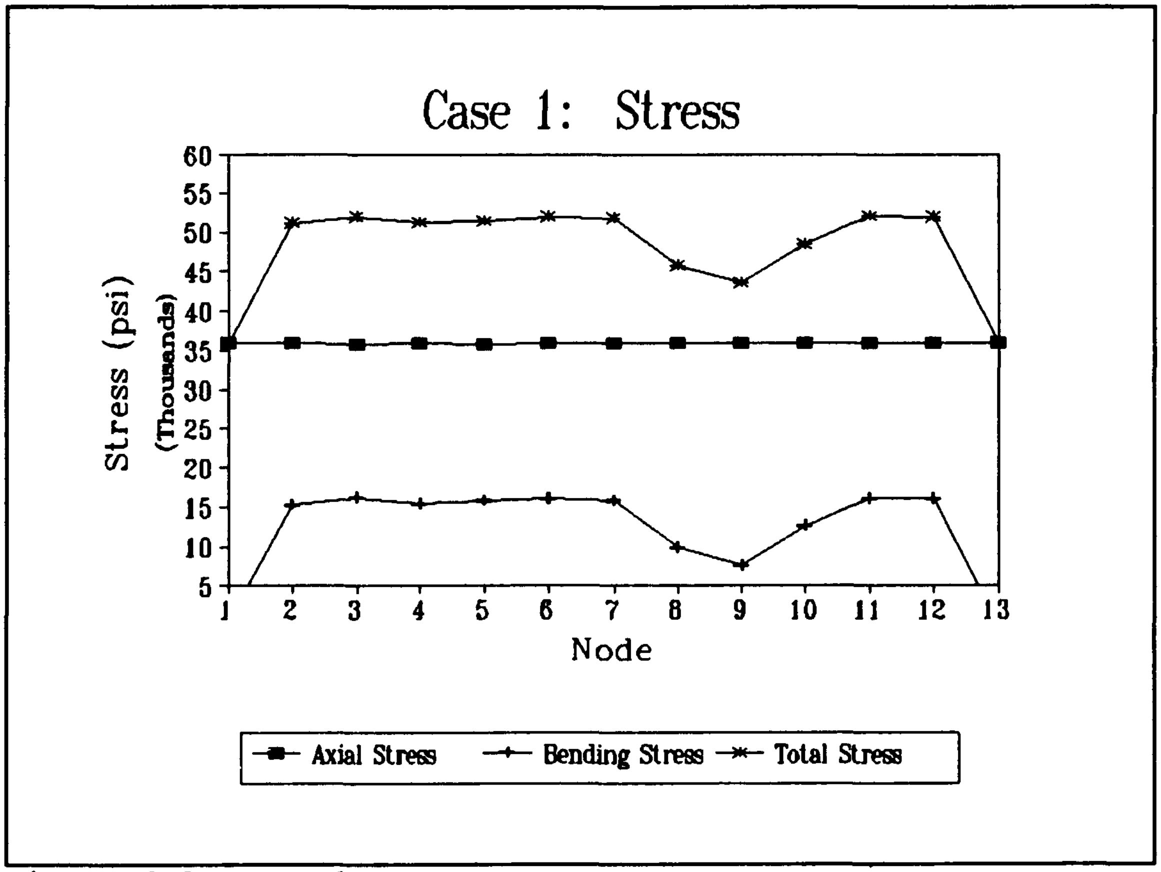

Figure 6.3: Case 1 - Stress

B. CASE 1A: HINGED-HINGED BEAM WITH DISTRIBUTED LOAD

This case is presented for comparison to Case 1. The slopes of the elements are not allowed to vary, so the arch remains a straight beam. There are seven active stress constraints and no active side constraints. The bending stresses dominate and the axial stresses are negligible. The volume of this arch is more than six times greater than that for Case 1.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 68.387 cubic inches

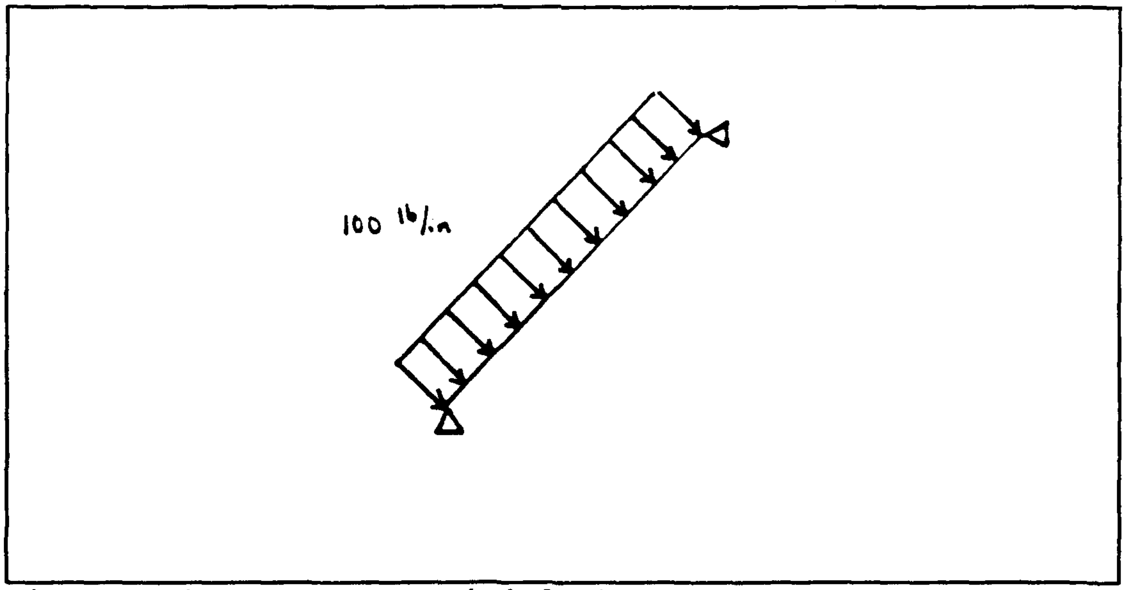

Figure 6.4: Case 1A - Initial Shape

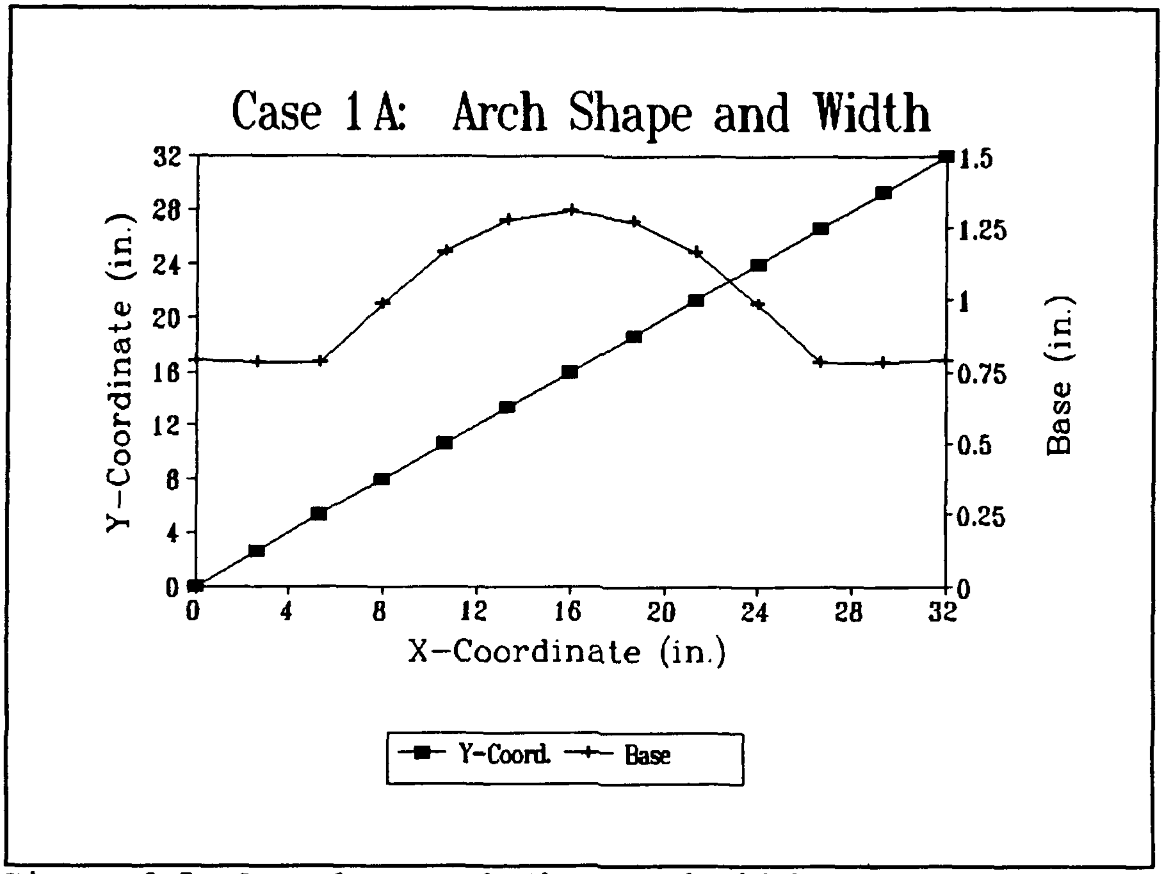

Figure 6.5: Case 1A - Arch Shape and Width

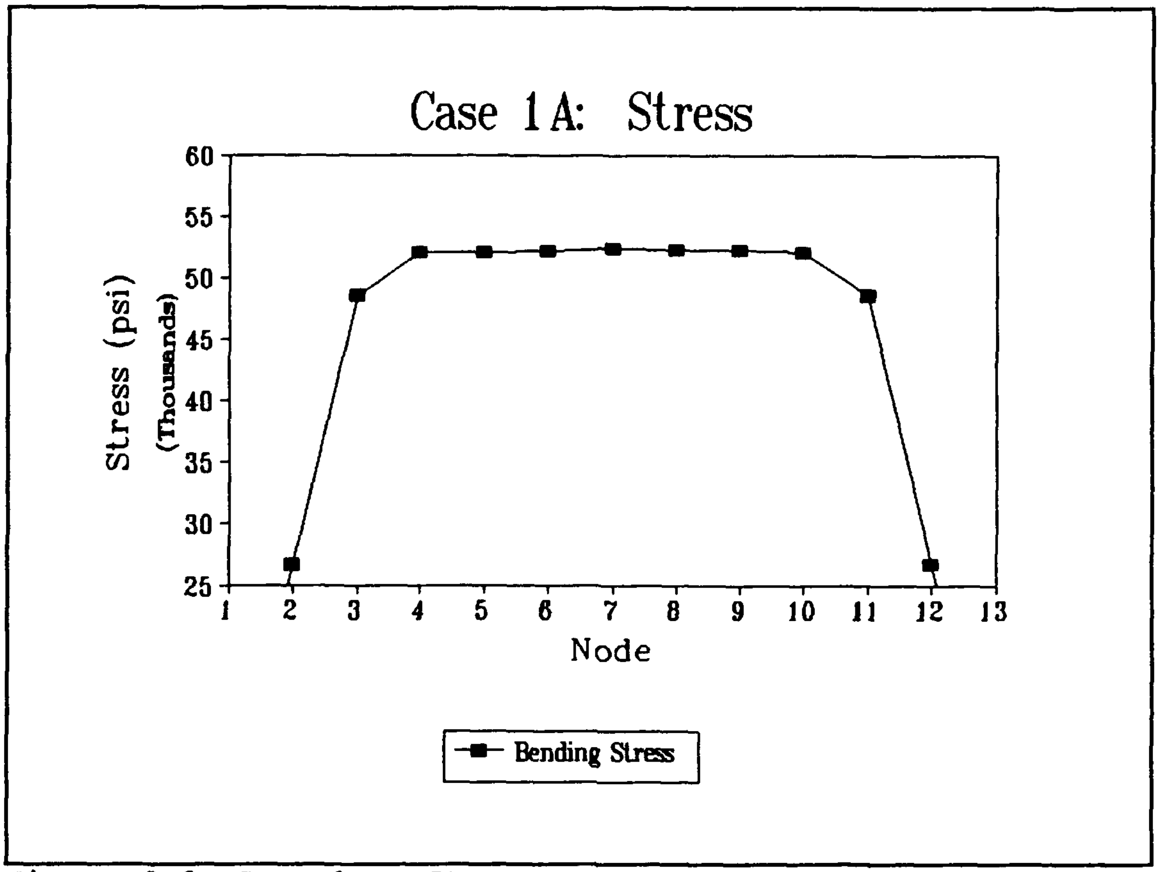

Figure 6.6: Case 1A - Stress

C. CASE 2: HINGED-HINGED ARCH WITH CONCENTRATED LOAD

This arch has a concentrated vertical load applied downward at node 7. There are seven active stress constraints and thirteen active side constraints (lower limit on base). The arch has taken the shape of two straight lines meeting at the center node. At eight of the thirteen nodes the axial stress dominates.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Concentrated Load = 2000 pounds (downward)
- Volume = 10.239 cubic inches

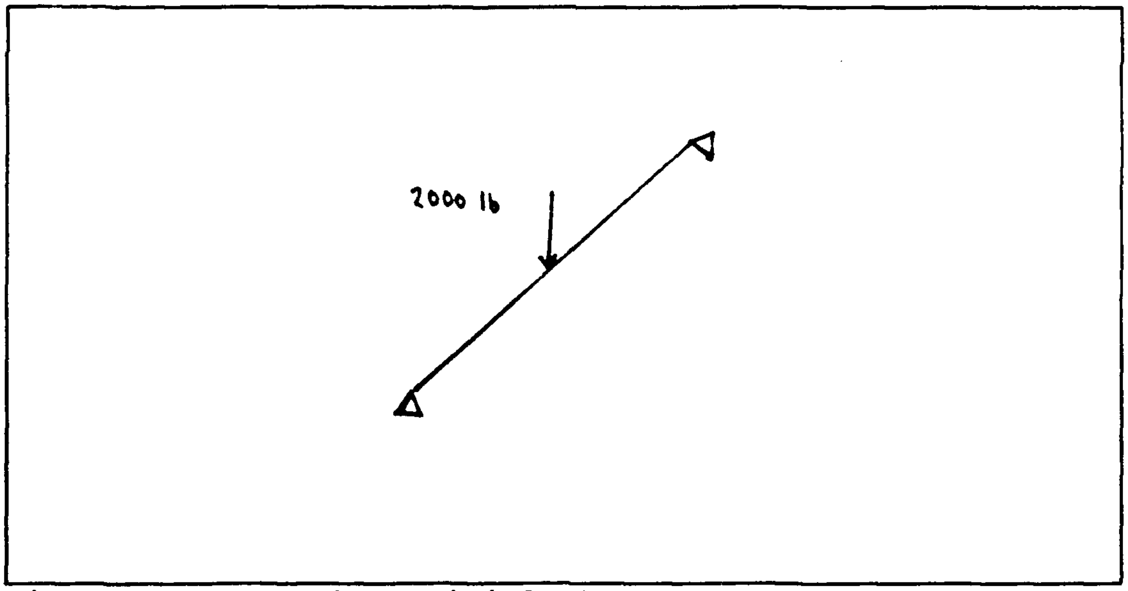

Figure 6.7: Case 2 - Initial Shape

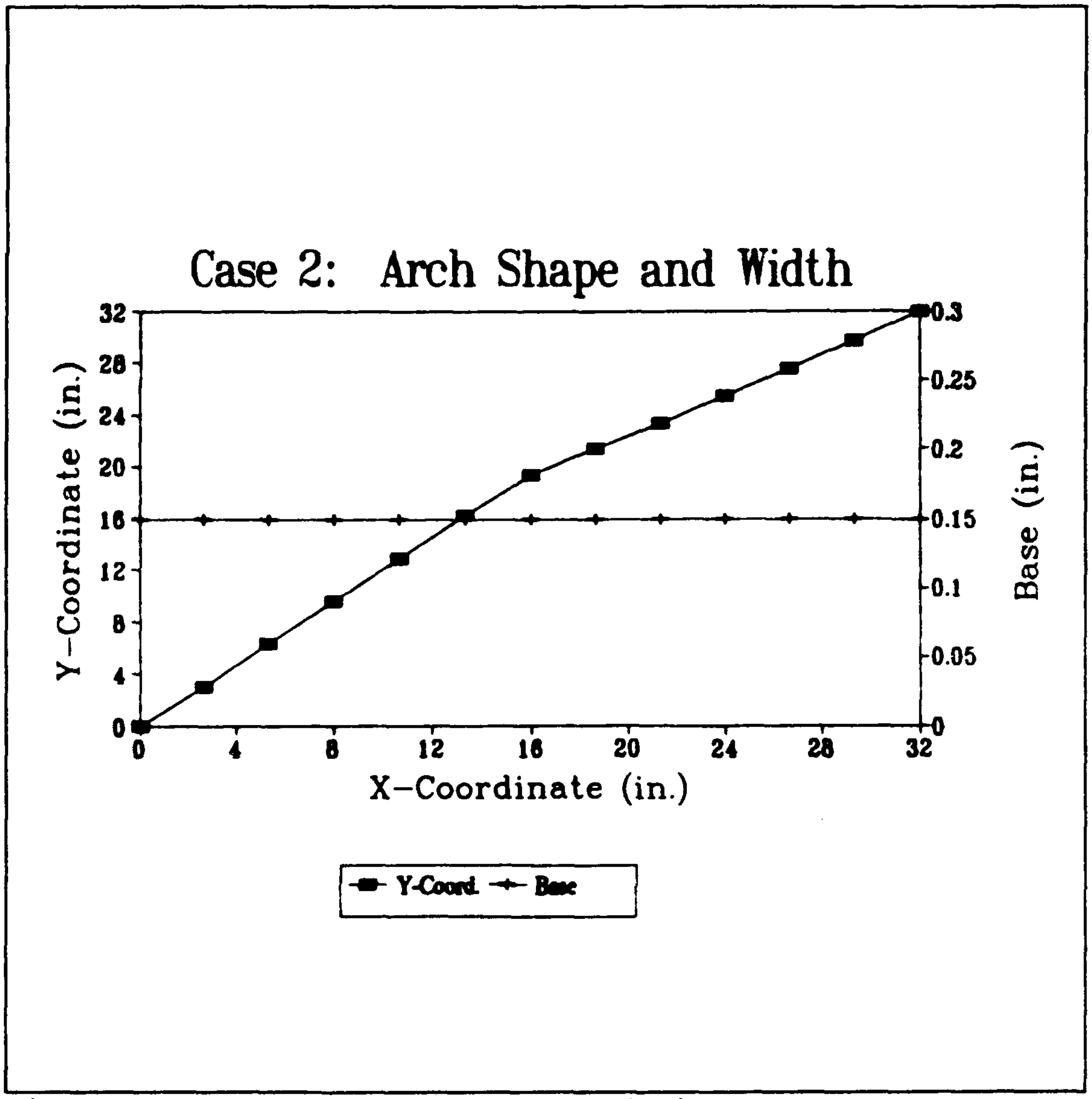

Figure 6.8: Case 2 - Arch Shape and Width

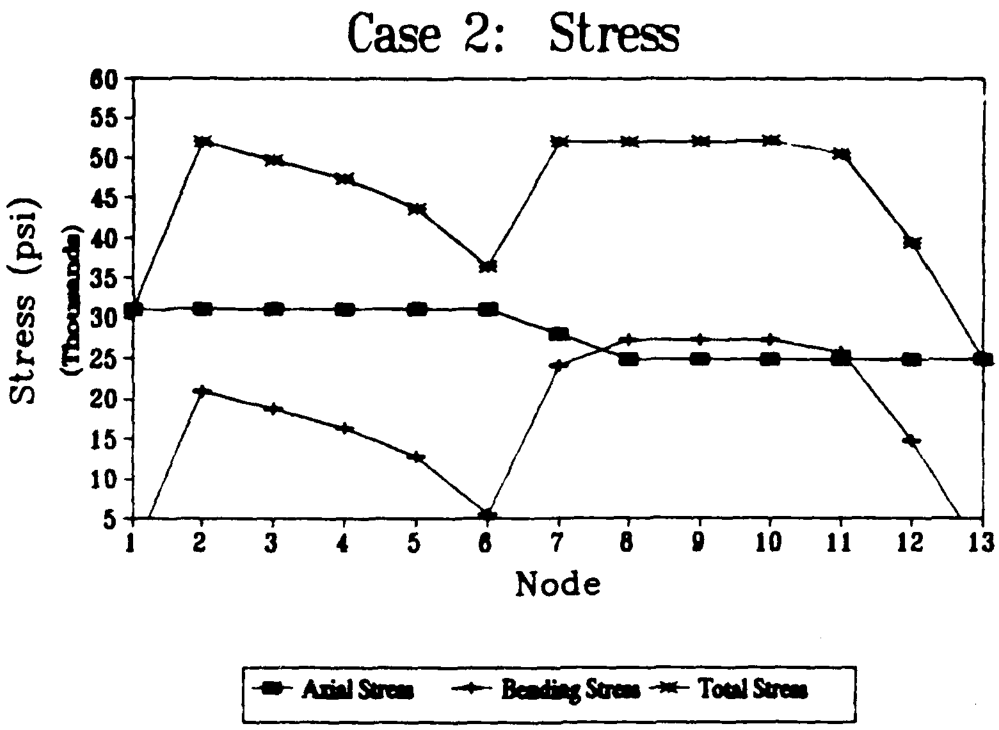

Figure 6.9: Case 2 - Stress

D. CASE 3: HINGED-HINGED ARCH WITH CONCENTRATED LOAD

This design has five active stress constraints and twelve active side constraints (lower limit on base). The arch has taken the shape of two straight lines meeting at node 7, where the concentrated load is applied. Once again, this design shows a preference for axial loading over bending moments.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Concentrated Load = 2828 pounds (applied down and to the right)
- Volume = 10.842 cubic inches

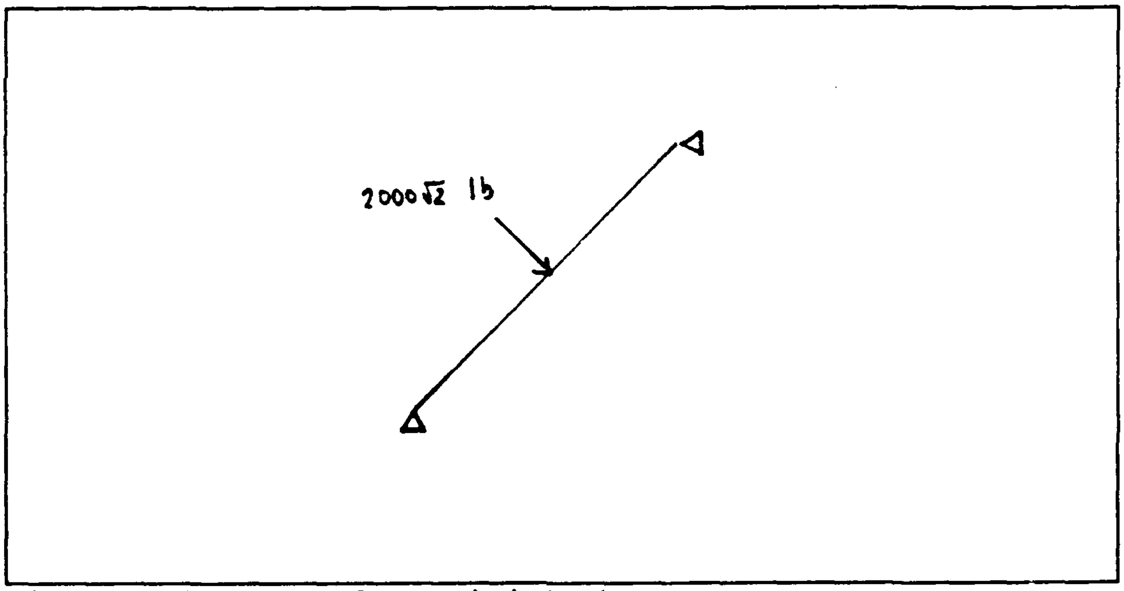

Figure 6.10: Case 3 - Initial Shape

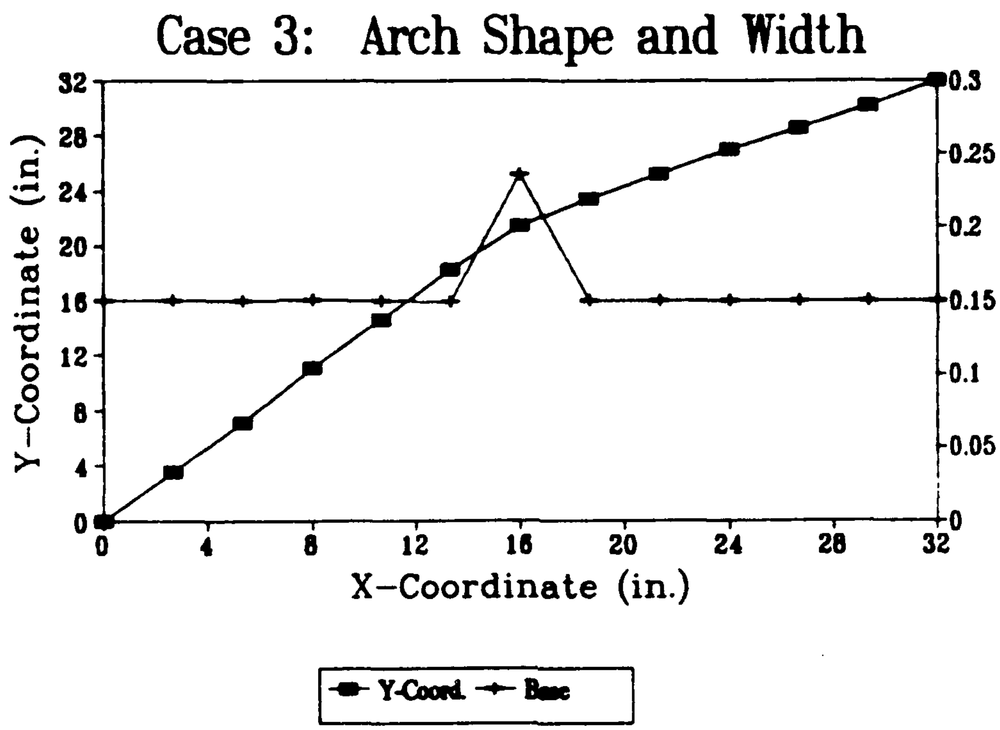

Figure 6.11: Case 3 - Arch Shape and Width

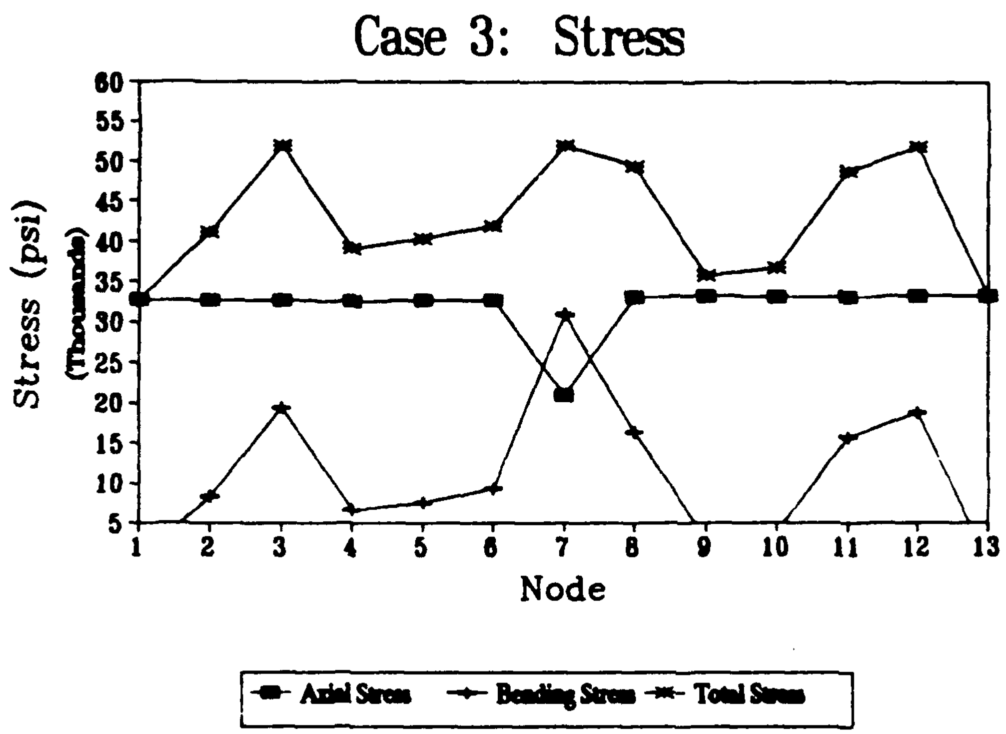

Figure 6.12: Case 3 - Stress

E. CASE 4: HINGED-HINGED ARCH WITH CONCENTRATED MOMENT

This design has one active stress constraint and twelve active side constraints (lower limit on base). The arch assumes the shape of a straight line, which is the minimum shape for a feasible design. The maximum base is at node 7, where the concentrated moment is applied.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Concentrated Moment = 6000 pound-inches
- Volume = 10.207 cubic inches

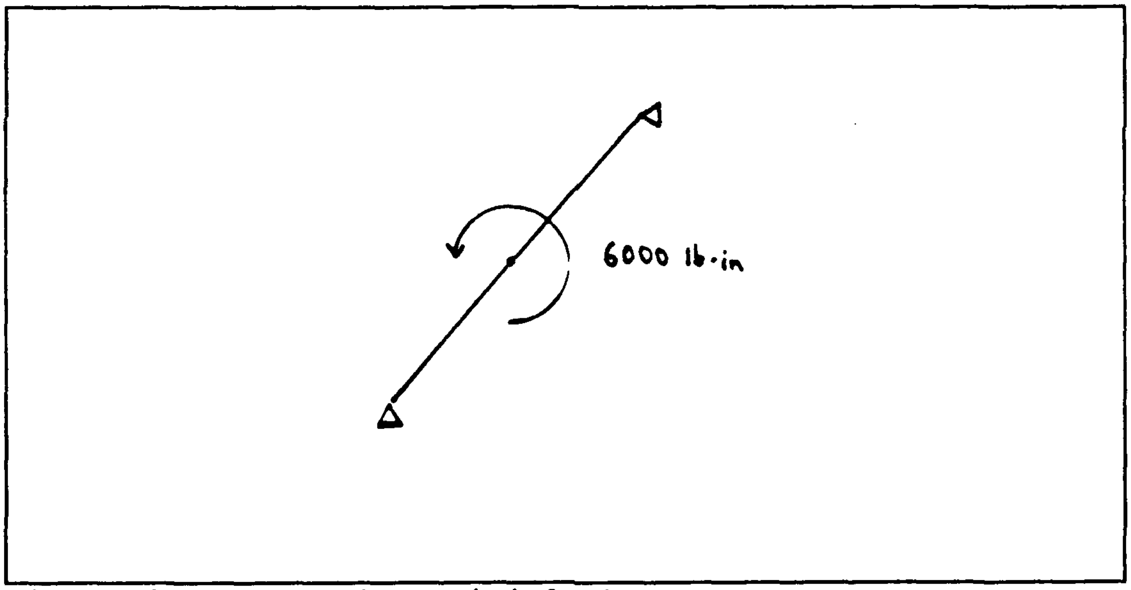

Figure 6.13: Case 4 - Initial Shape

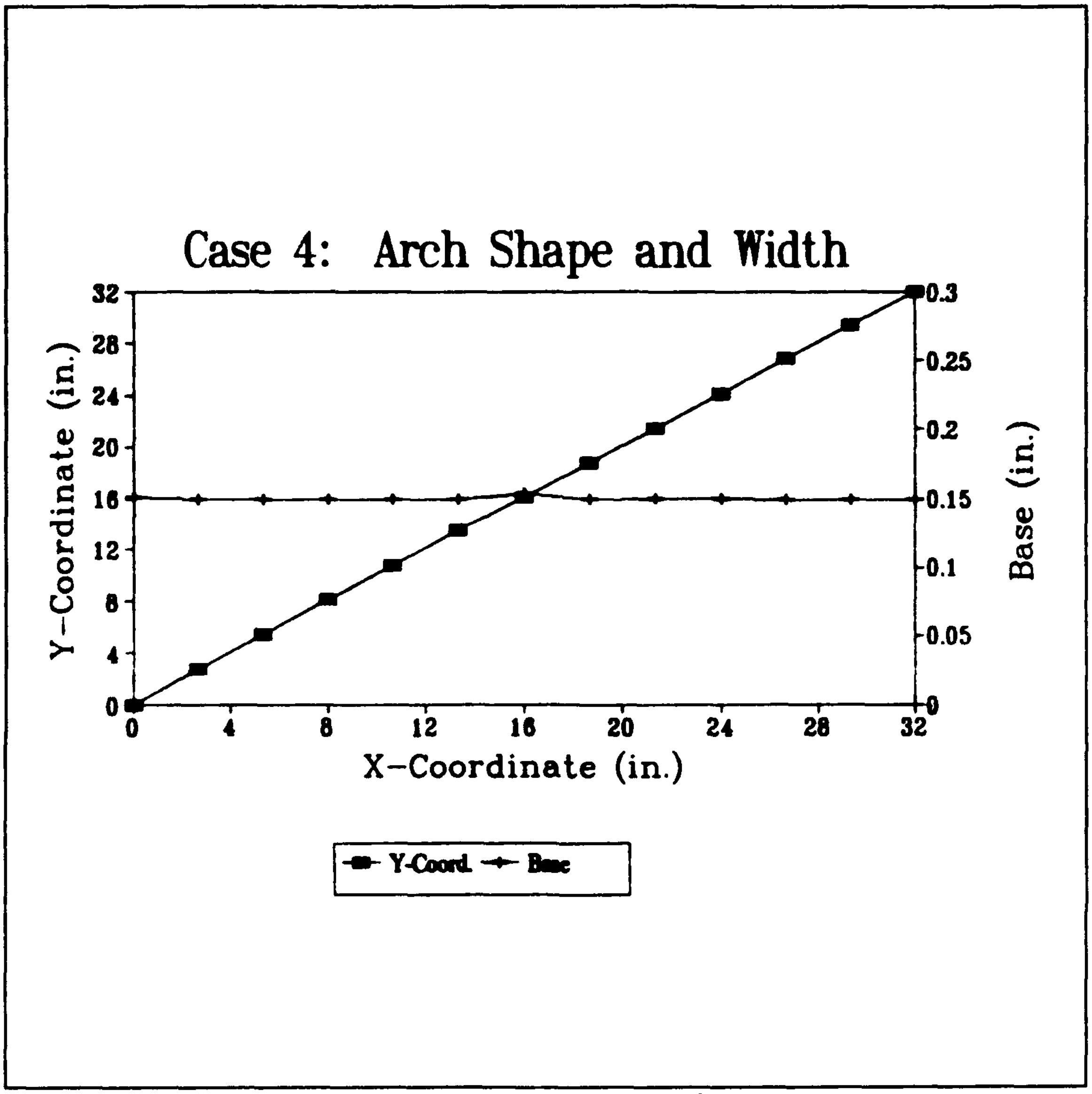

Figure 6.14: Case 4 - Arch Shape and Width

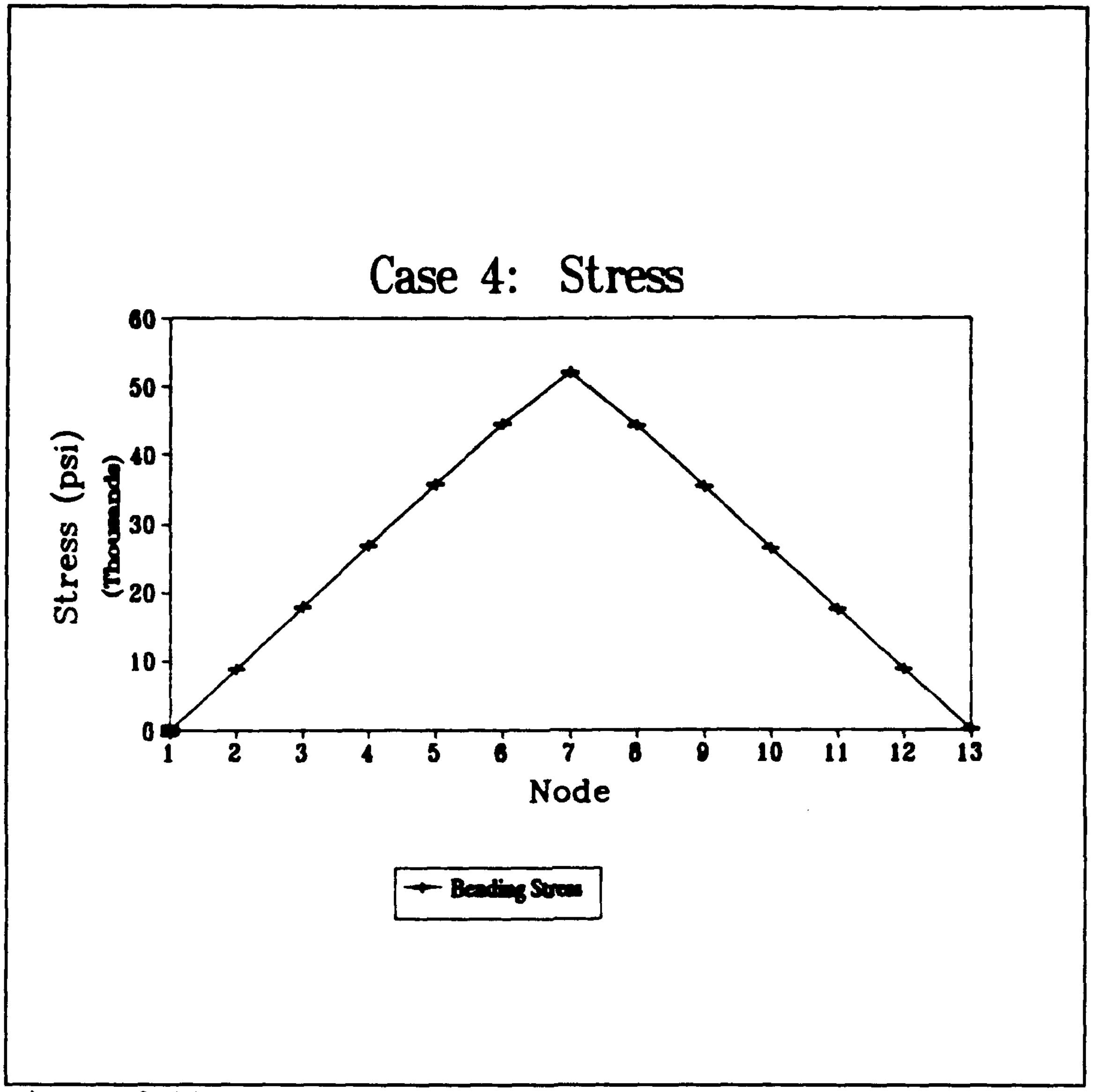

Figure 6.15: Case 4 - Stress

F. CASE 5: ROLLER-HINGED ARCH WITH DISTRIBUTED LOAD

This design has ten active stress constraints and twelve active side constraints (lower limit on base). The arch has assumed a nearly circular shape. In this case, the bending stress dominates throughout the arch. It appears that the arch has attained the maximum degree of axial loading that can be achieved with these boundary conditions.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 11.329 cubic inches

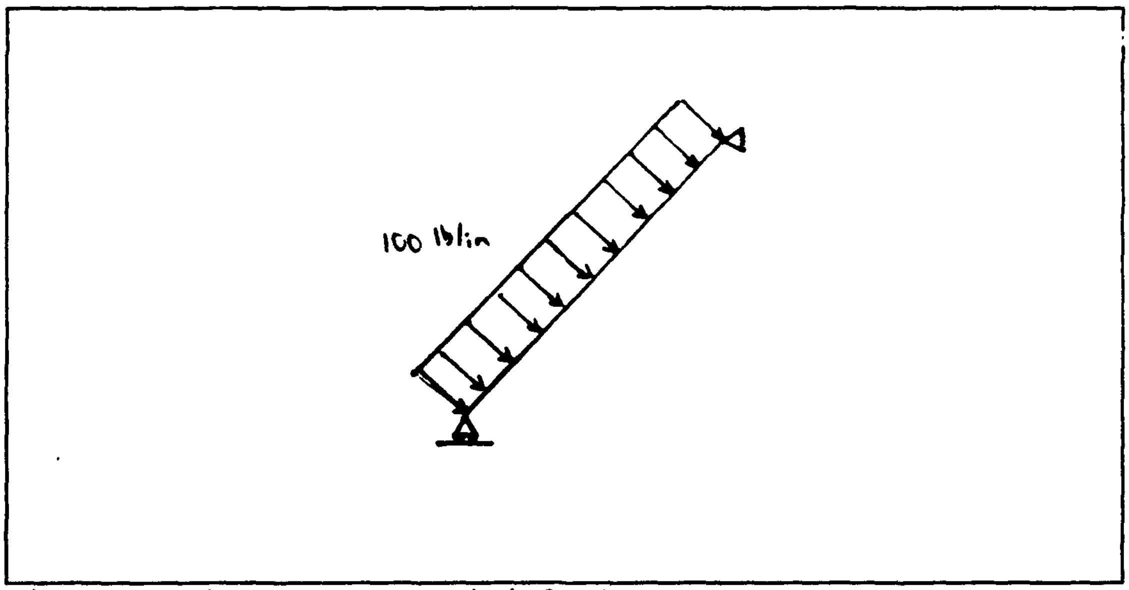

Figure 6.16: Case 5 - Initial Shape

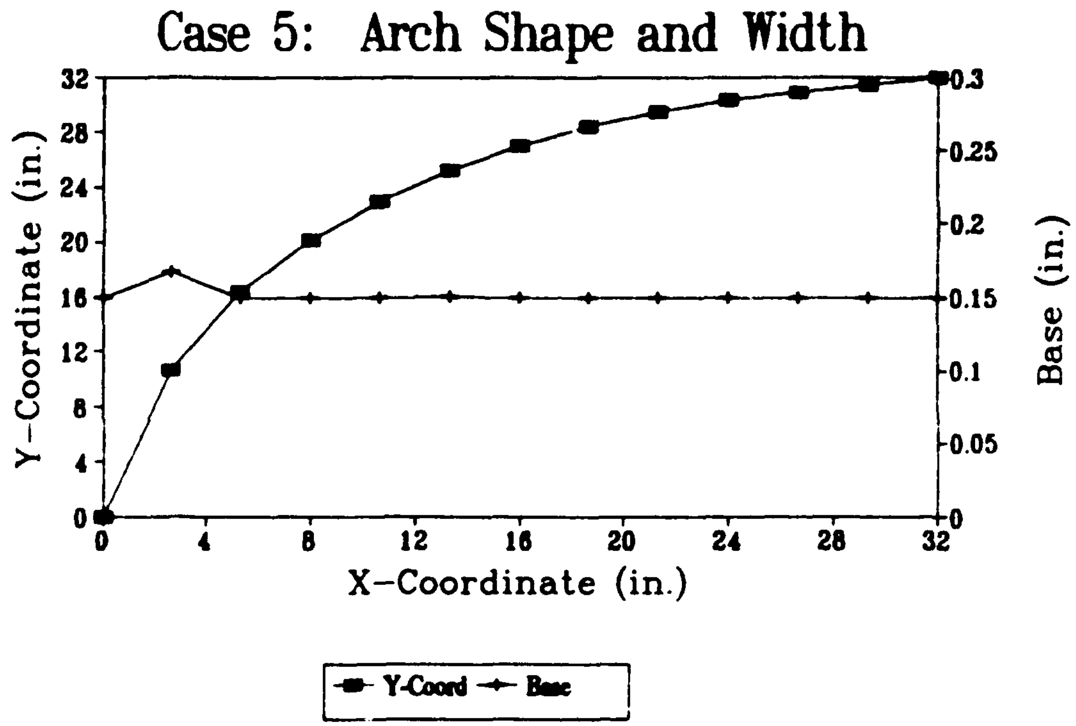

Figure 6.17: Case 5 - Arch Shape and Width

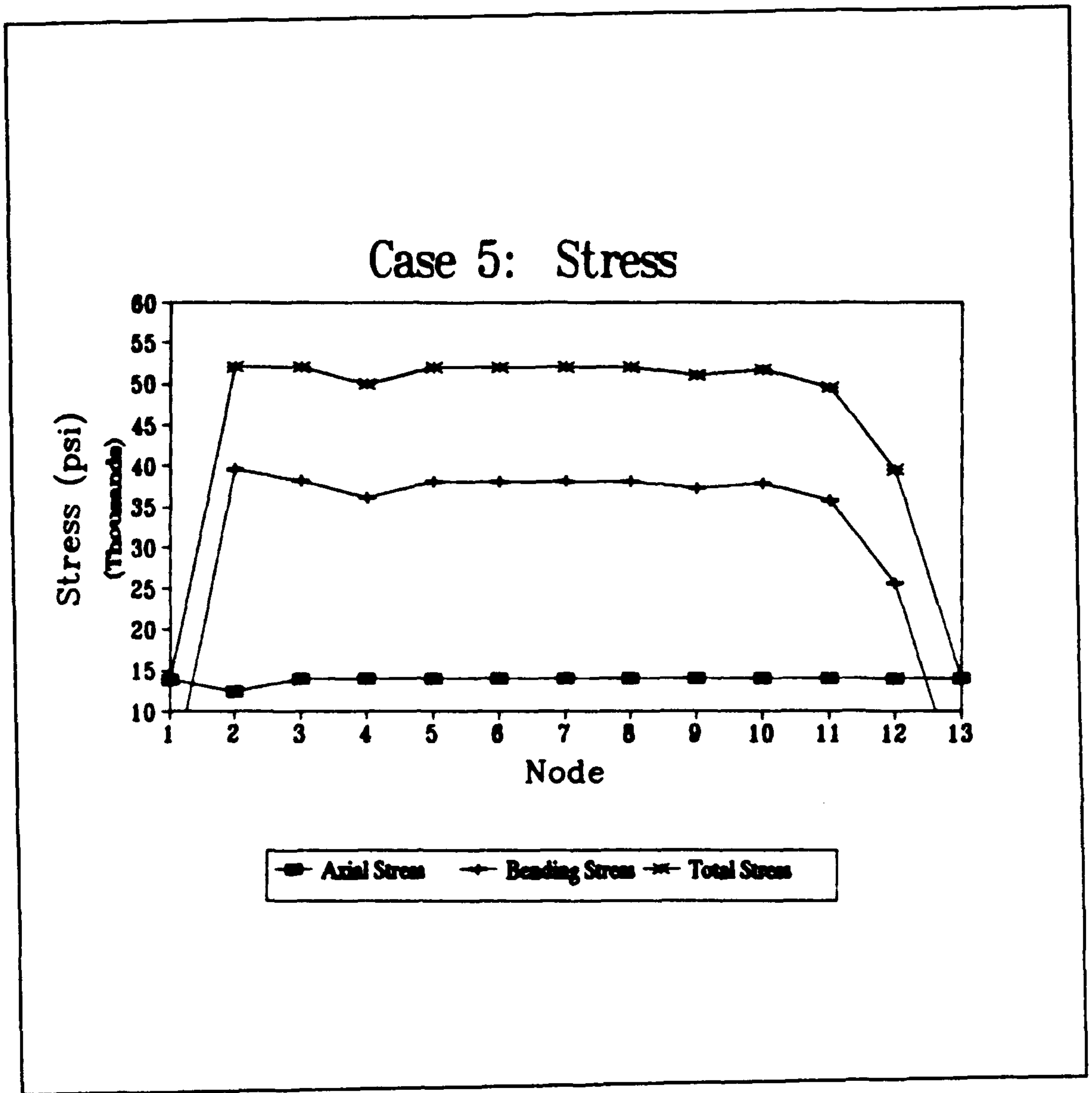

Figure 6.18: Case 5 - Stress

G. CASE 6: ROLLER-FIXED ARCH WITH DISTRIBUTED LOAD

This design has four active stress constraints and ten active side constraints (lower limit on base). The bending stresses dominated for the most part, but the axial stresses are significant. The maximum base is at the fixed end, as expected.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 14.431 cubic inches

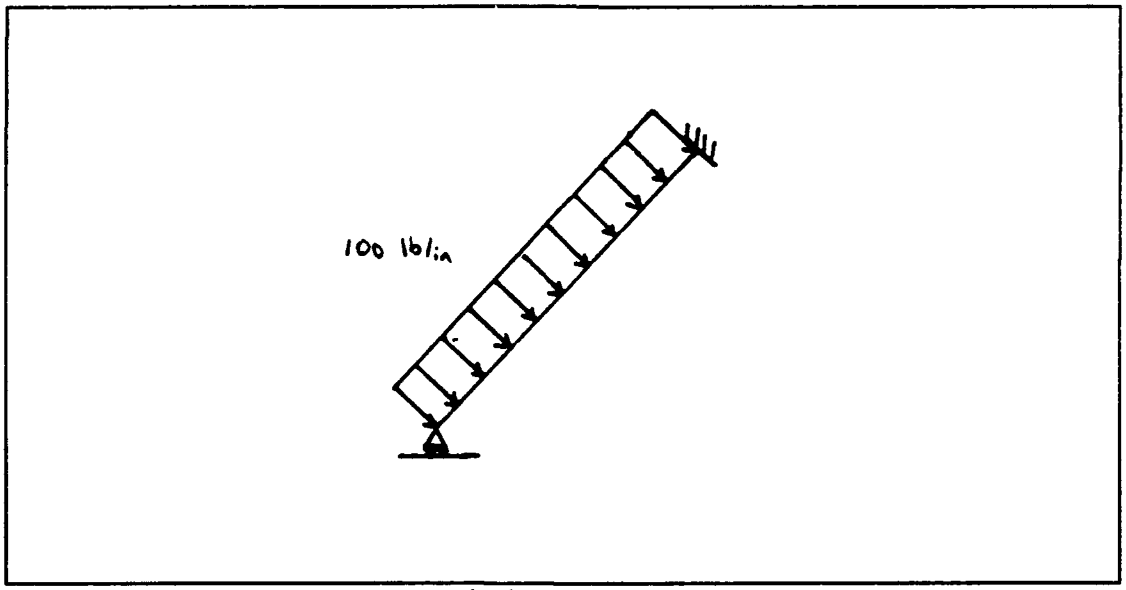

Figure 6.19: Case 6 - Initial Shape

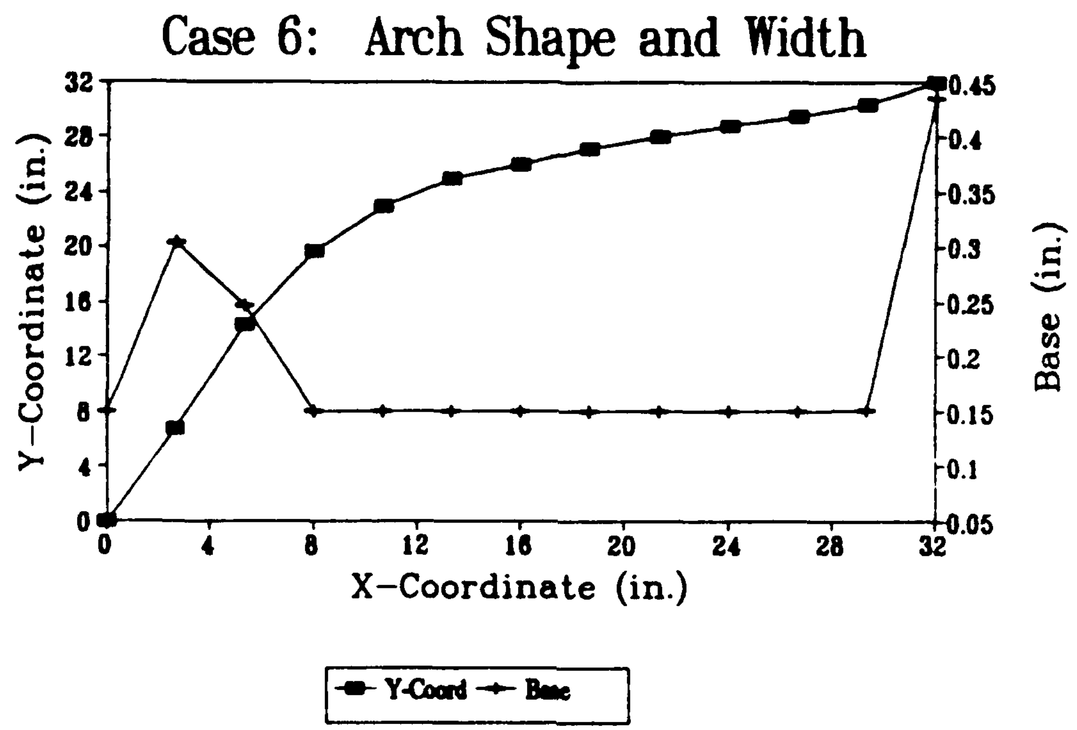

Figure 6.20: Case 6 - Arch Shape and Width

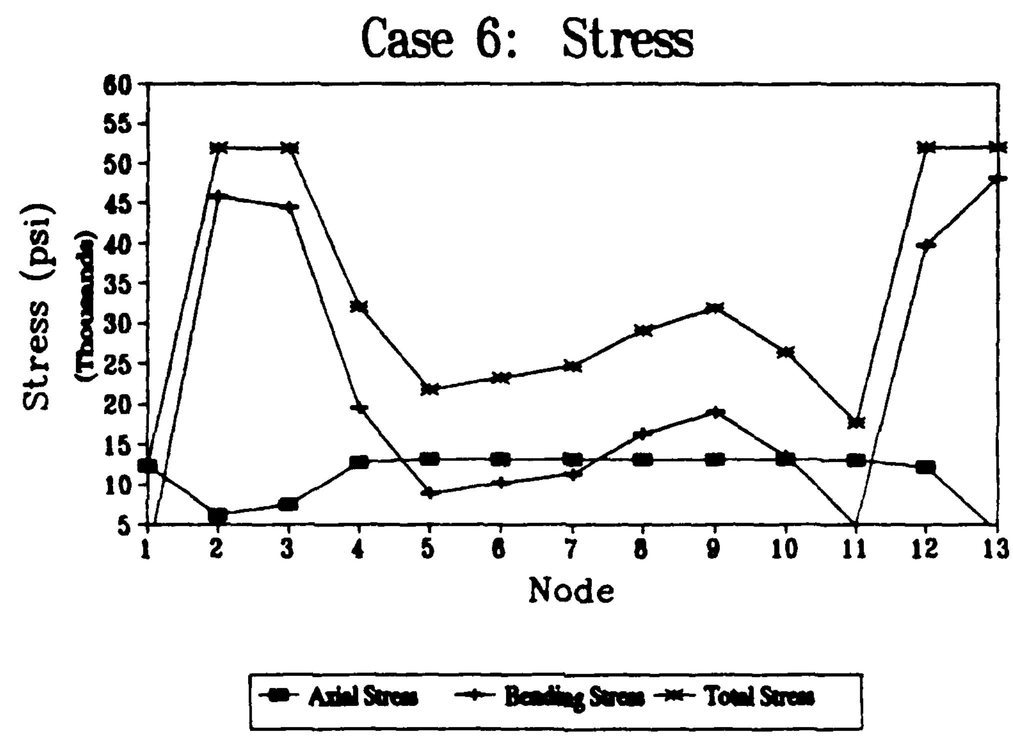

Figure 6.21: Case 6 - Stress

H. CASE 7: FIXED-FREE ARCH WITH DISTRIBUTED LOAD

Using the cross-sectional depth of the previous case studies resulted in a design that consistently violated the stress constraints at the nodes at and close to the fixed end of the arch. Therefore a cross-sectional depth of 3 inches was used. The resulting design has seven active stress constraints and six active side constraints (lower limit on base). The arch has taken the shape of a tapered beam with a maximum base at the fixed end. The bending stresses dominate and the axial loads are negligible. Because of the lack of support at the free end, the arch is unable to attain axial loading in the elements.

- H = 32.0 inches
- L = 32.0 inches
- h = 3.0 inches
- Distributed Load = -100 pounds/inch
- Volume = 72.614 cubic inches

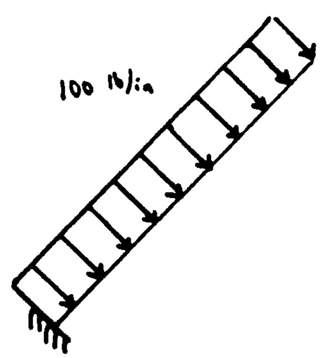

Figure 6.22: Case 7 - Initial Shape

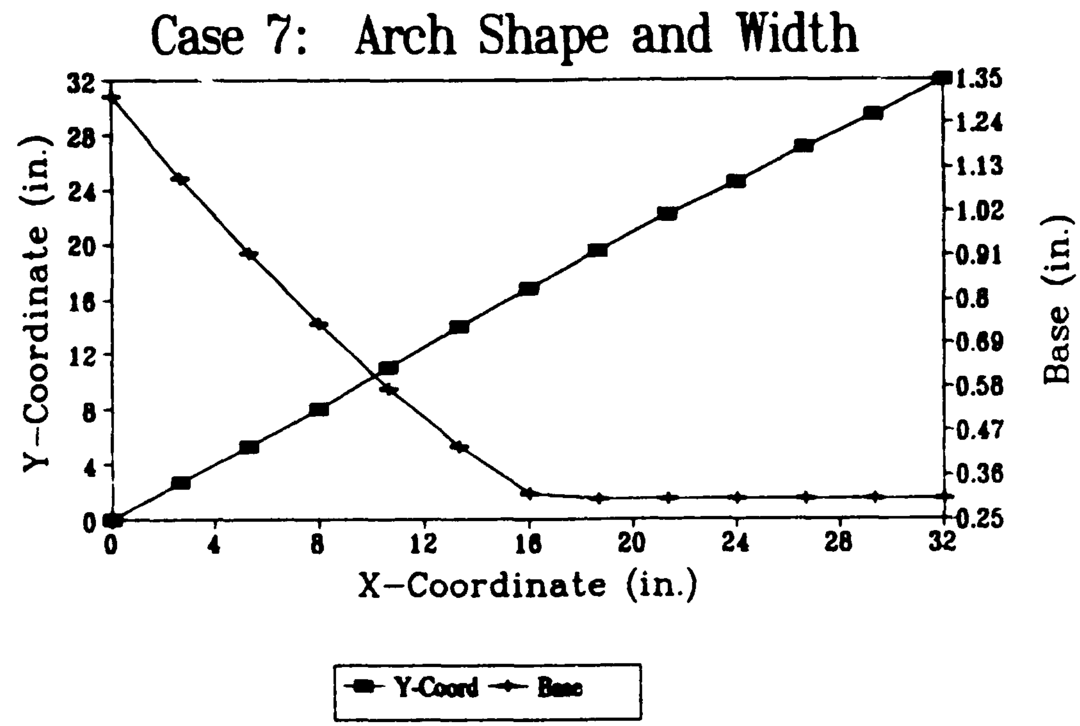

Figure 6.23: Case 7 - Arch Shape and Width

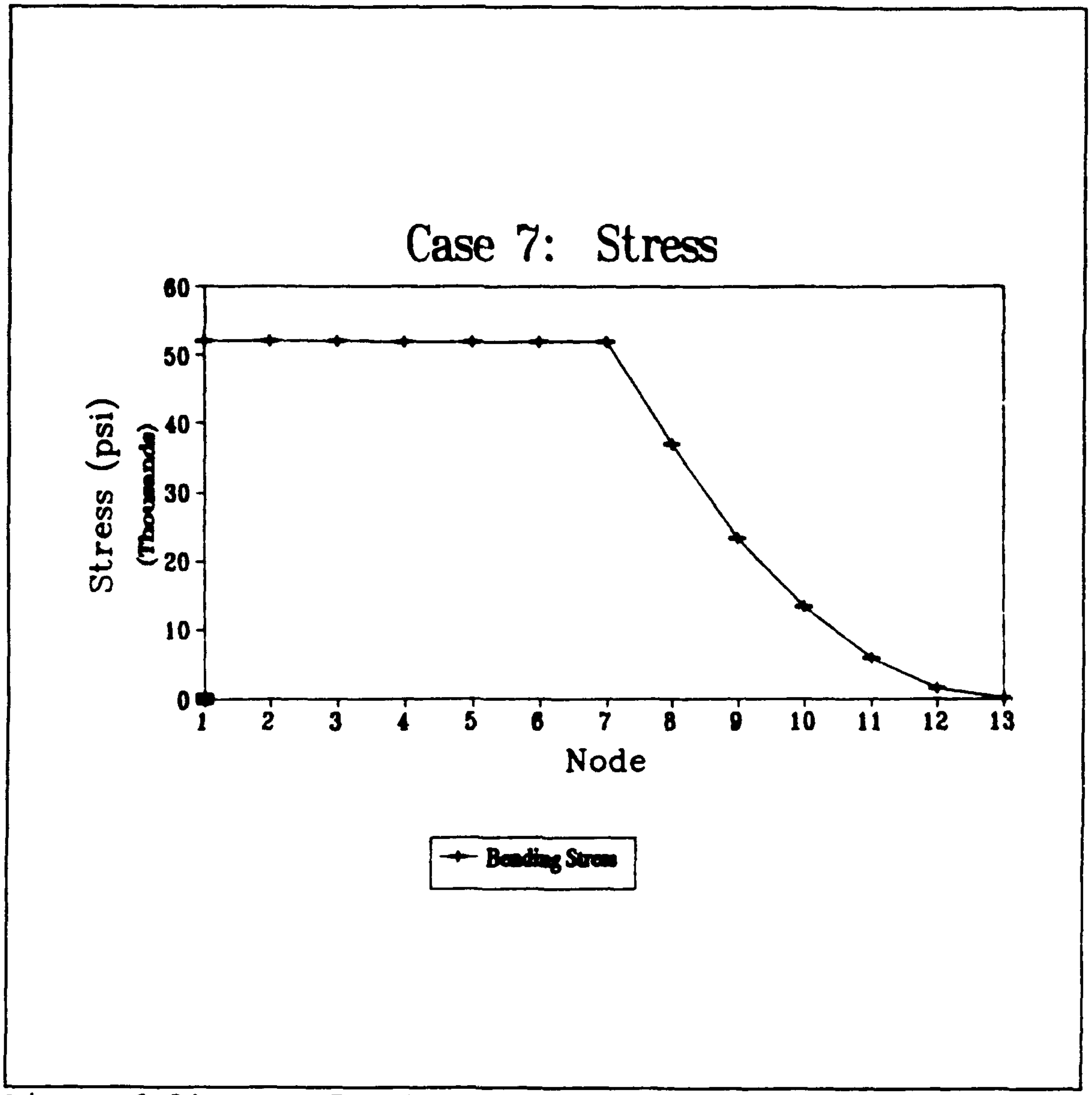

Figure 6.24: Case 7 - Stress

I. CASE 8: FIXED-FIXED ARCH WITH DISTRIBUTED LOAD

This design has six active stress constraints and eleven active side constraints. The arch curves outward slightly in the center. With the exception of the nodes at the ends, axial stresses dominate in the arch. The maximum base dimensions occur at the fixed ends.

- H = 32.0 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 11.308 inches

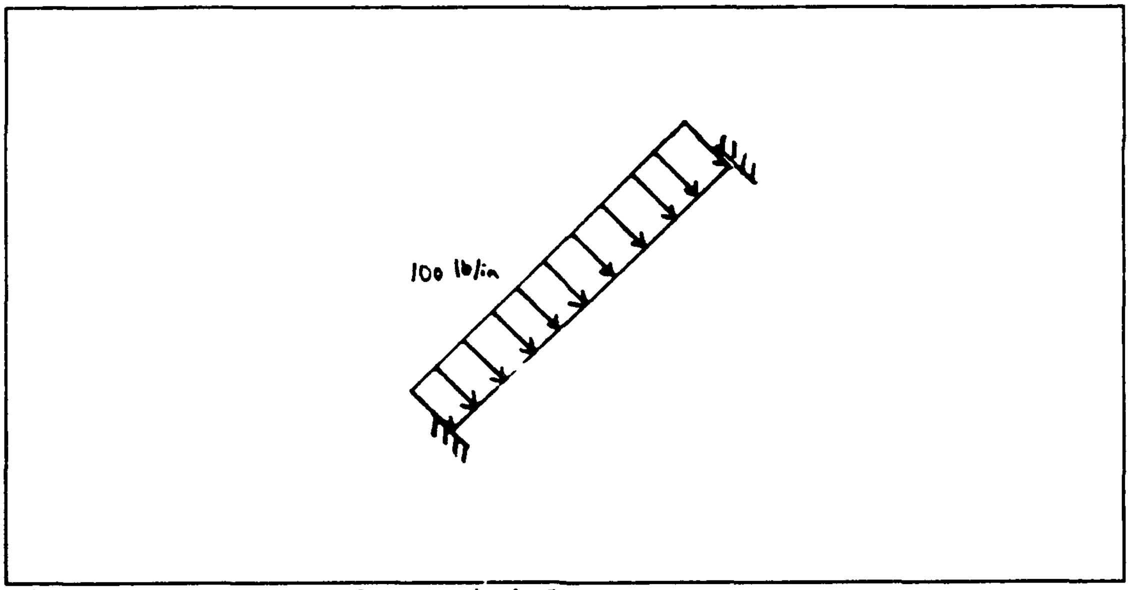

Figure 6.25: Case 8 - Initial Shape

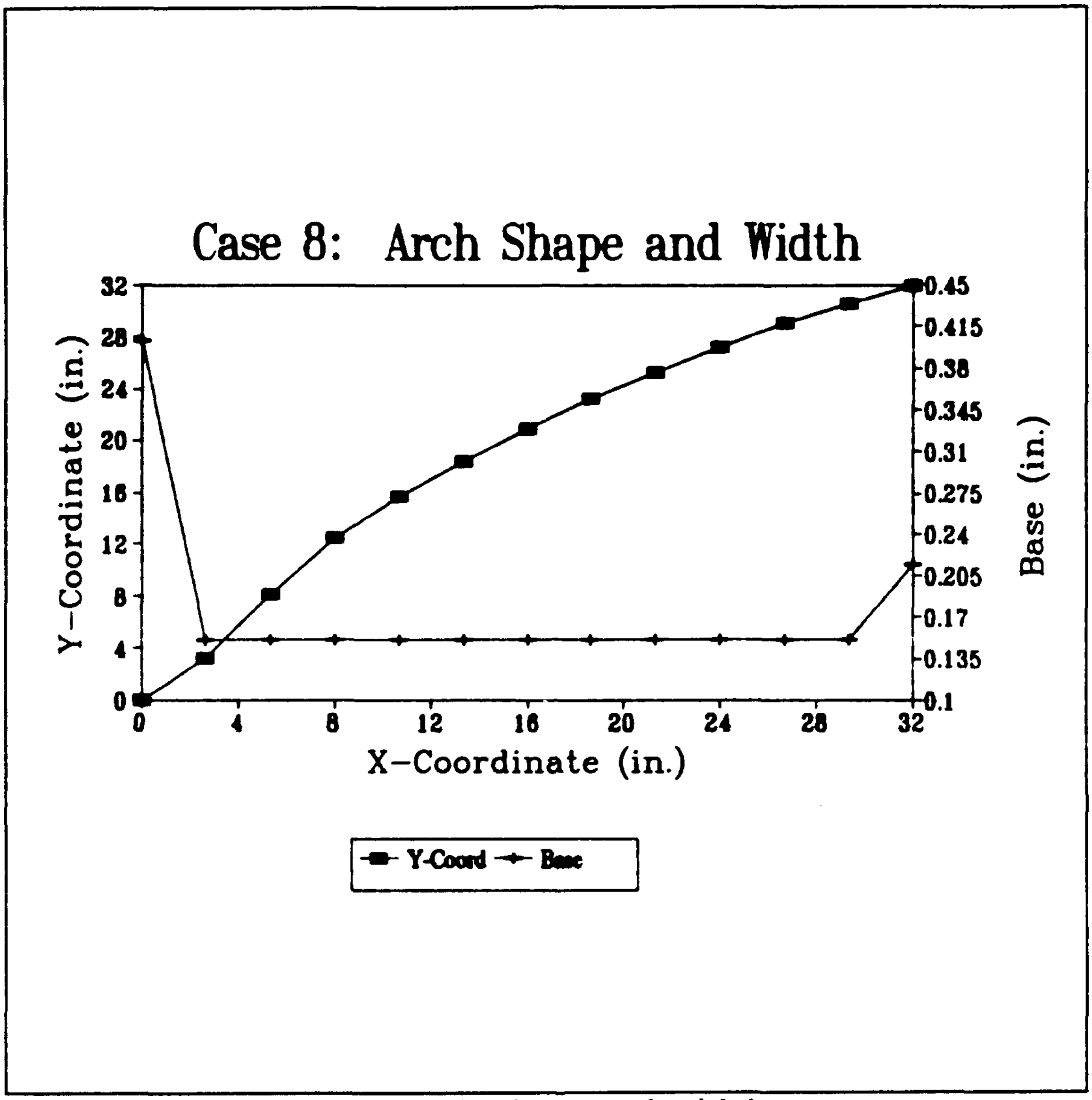

Figure 6.26: Case 8 - Arch Shape and Width

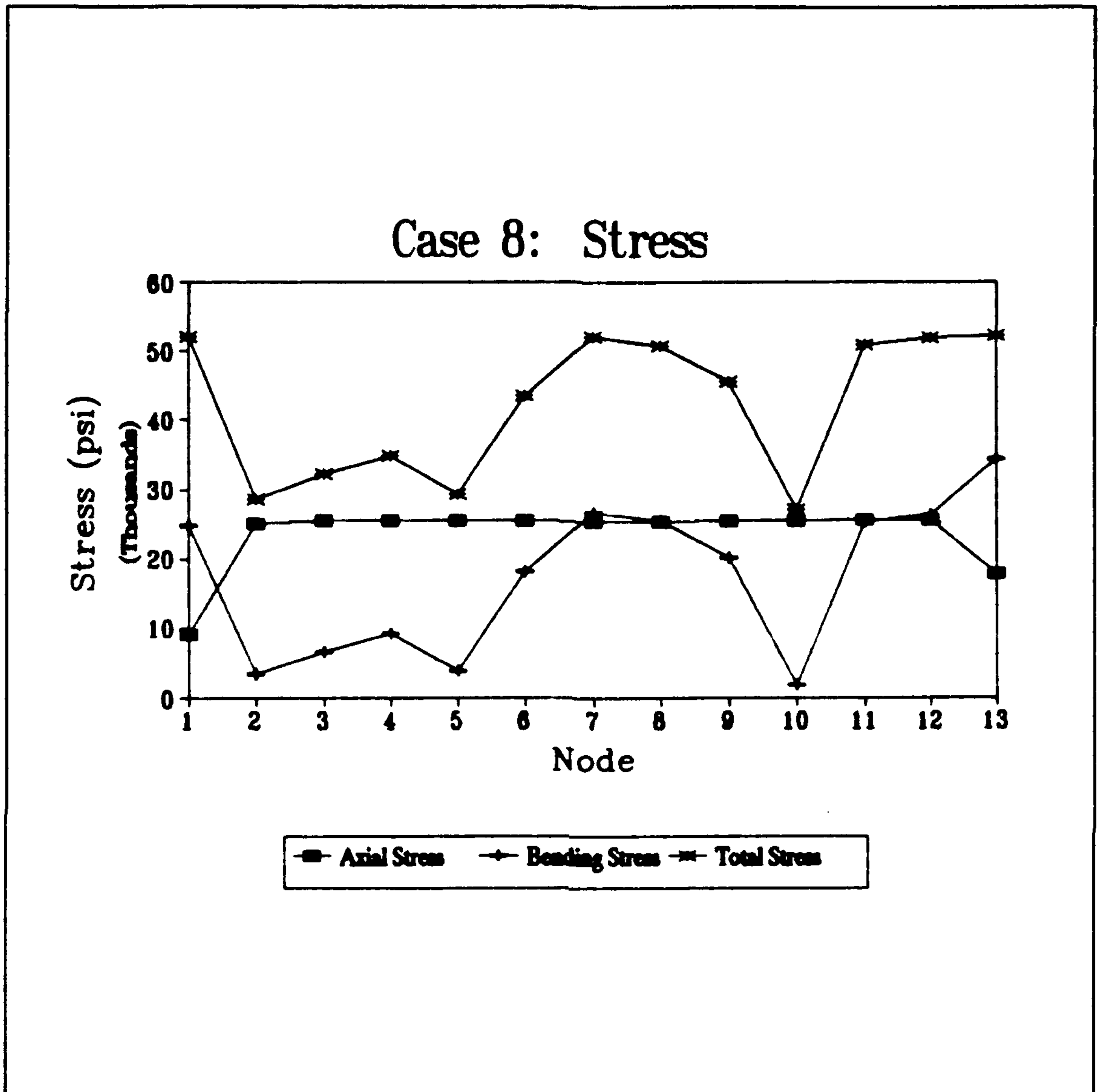

Figure 6.27: Case 8 - Stress

J. CASE 9: HINGED-HINGED ARCH WITH DISTRIBUTED LOAD

This case differs from Case 1 only in the distance spanned in the vertical direction , H. In this case, H = 18.475 inches. There are eleven active stress constraints and thirteen active side constraints (lower limit on base). The arch has assumed a curved shape similar to Case 1. Once again, there is a clear preference for axial loading.

- H = 18.475 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 8.391 cubic inches

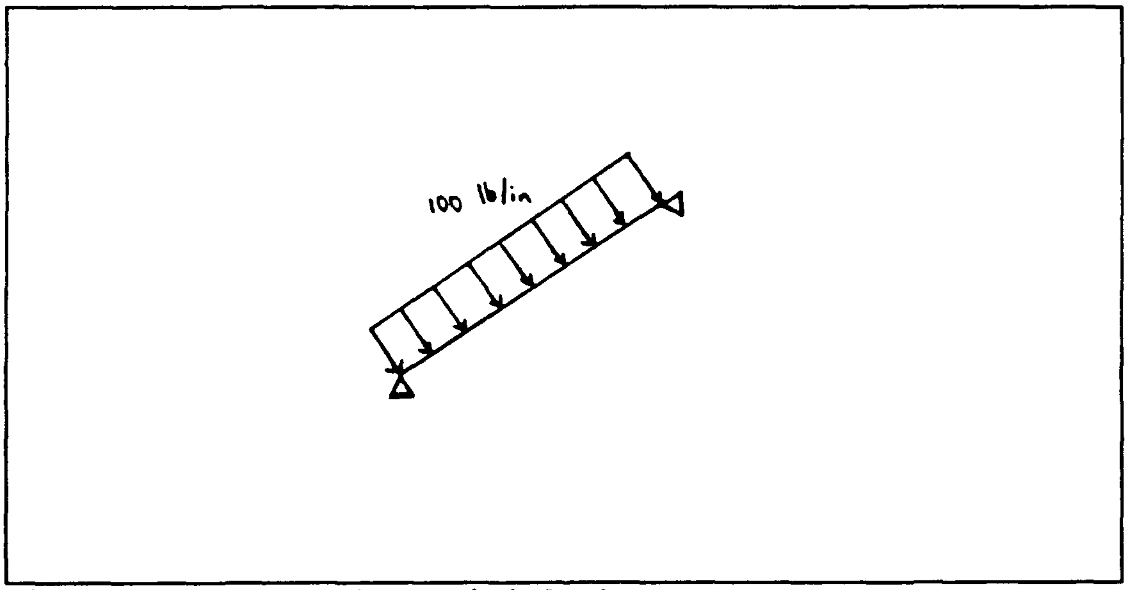

Figure 6.28: Case 9 - Initial Shape

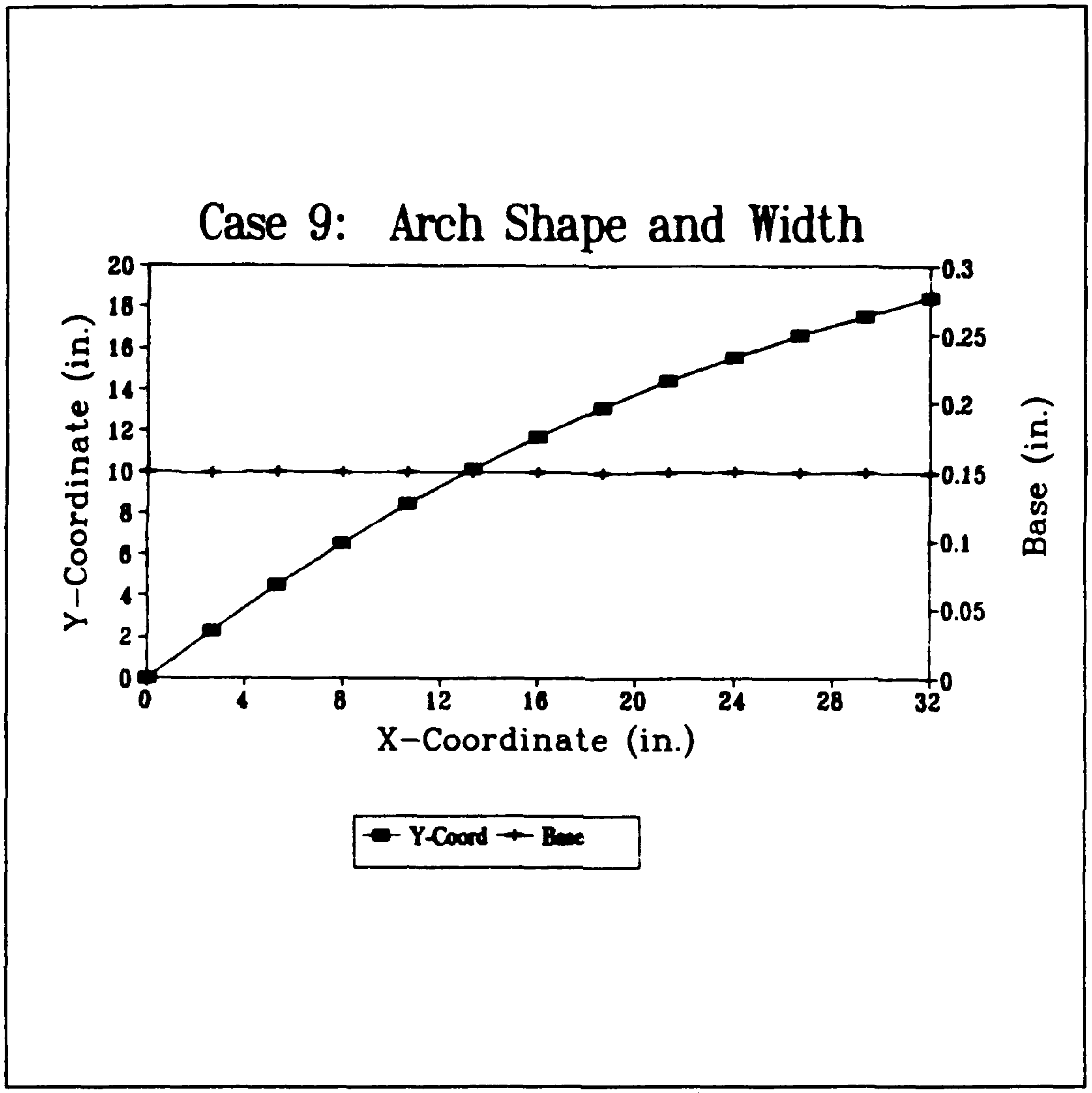

Figure 6.29: Case 9 - Arch Shape and Width

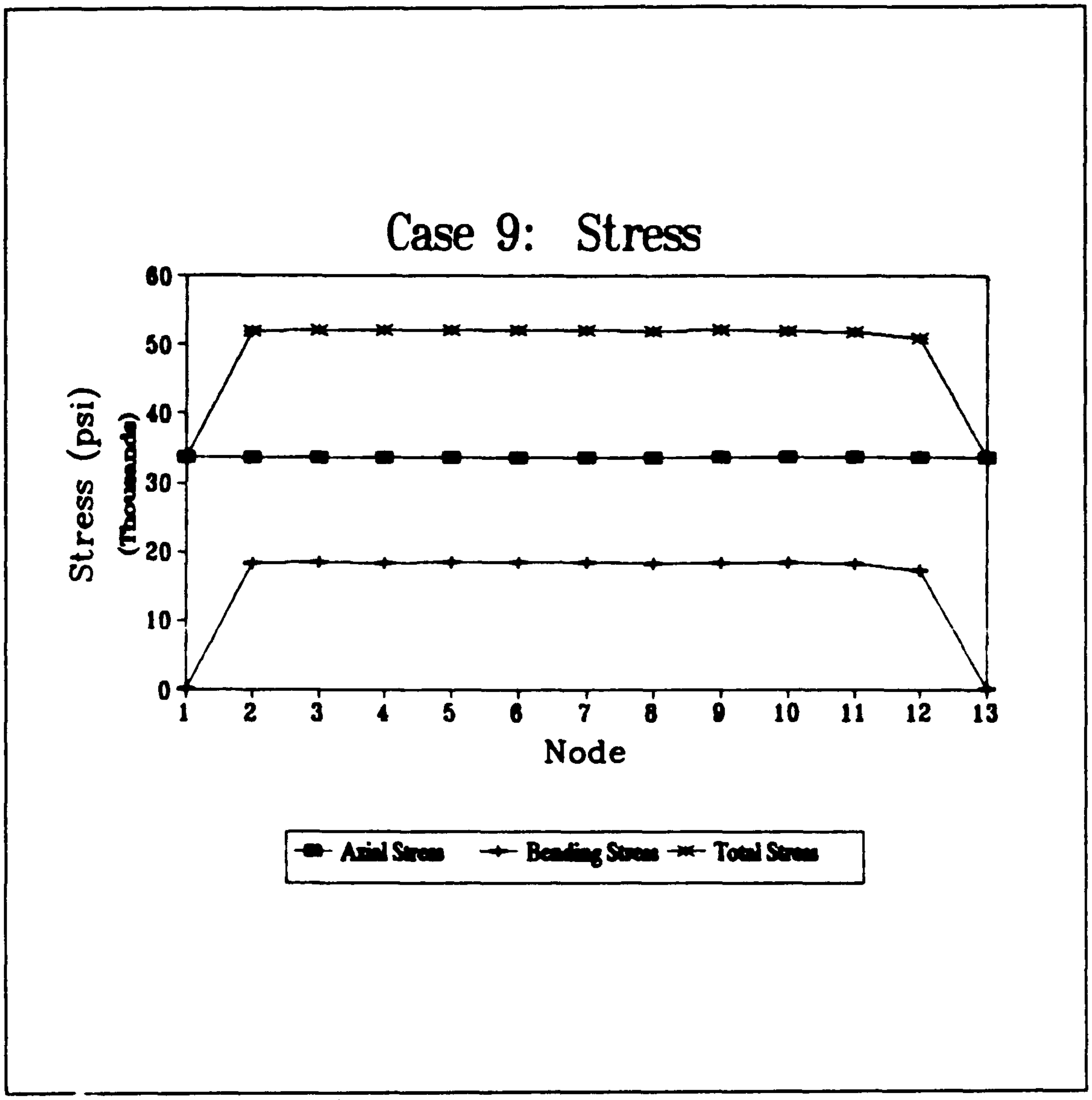

Figure 6.30: Case 9 - Stress

K. CASE 10: HINGED-HINGED ARCH WITH DISTRIBUTED LOAD

This case differs from the previous case only in the distance spanned in the vertical direction, H. There are eight active stress constraints and twelve active side constraints. Once again, the arch curves outward and the axial stresses dominate.

- H = 55.426 inches
- L = 32.0 inches
- h = 1.5 inches
- Distributed Load = -100 pounds/inch
- Volume = 15.149 cubic inches

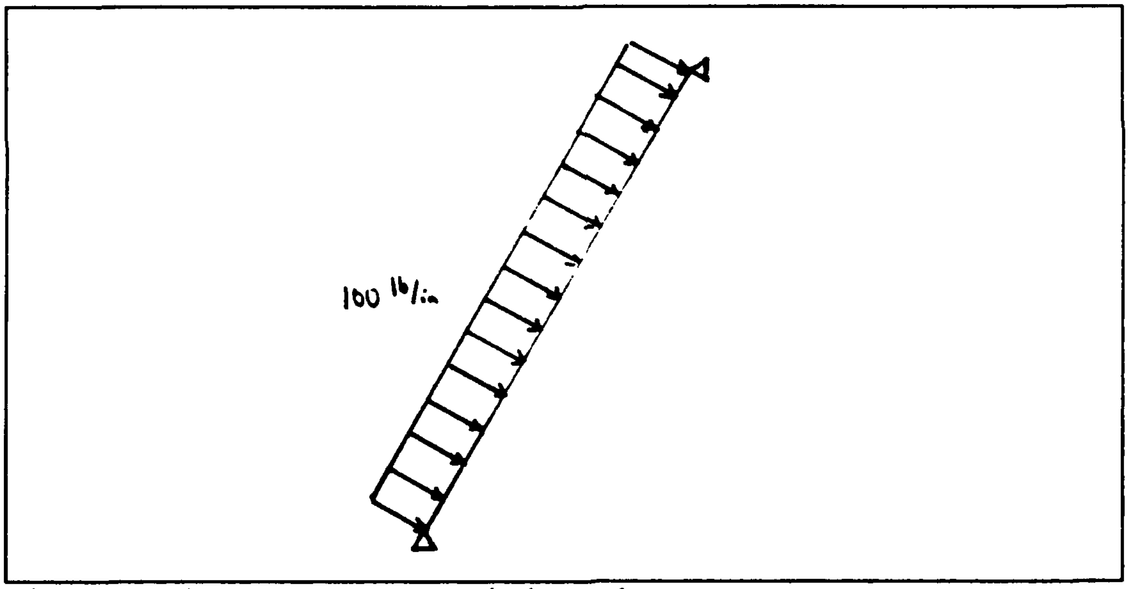

Figure 6.31: Case 10 - Initial Shape

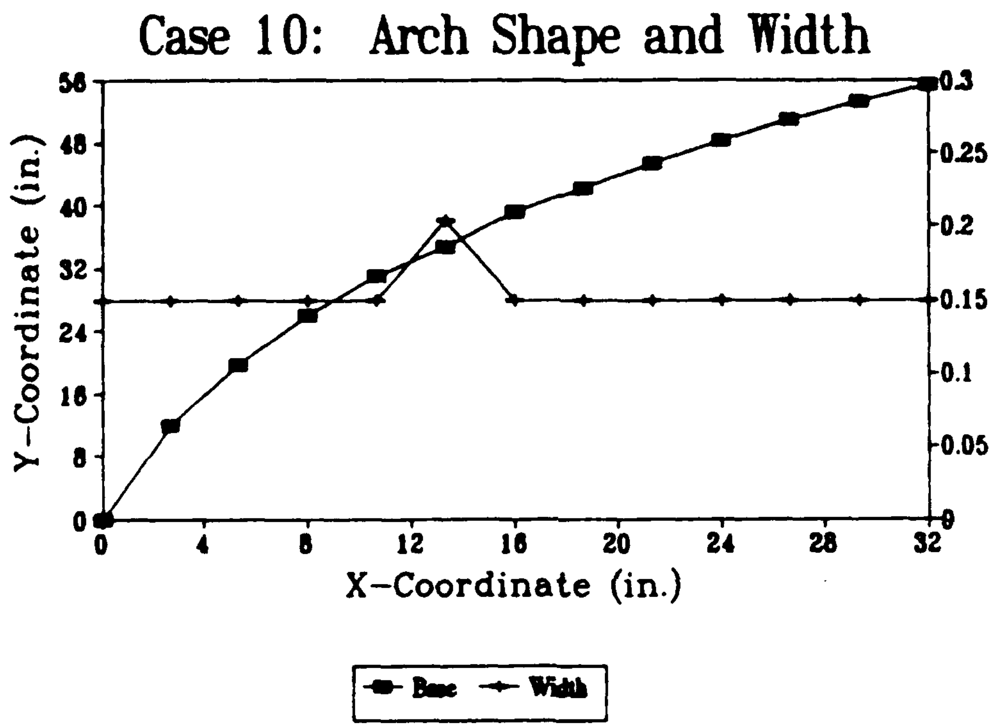

Figure 6.32: Case 10 - Arch Shape and Width

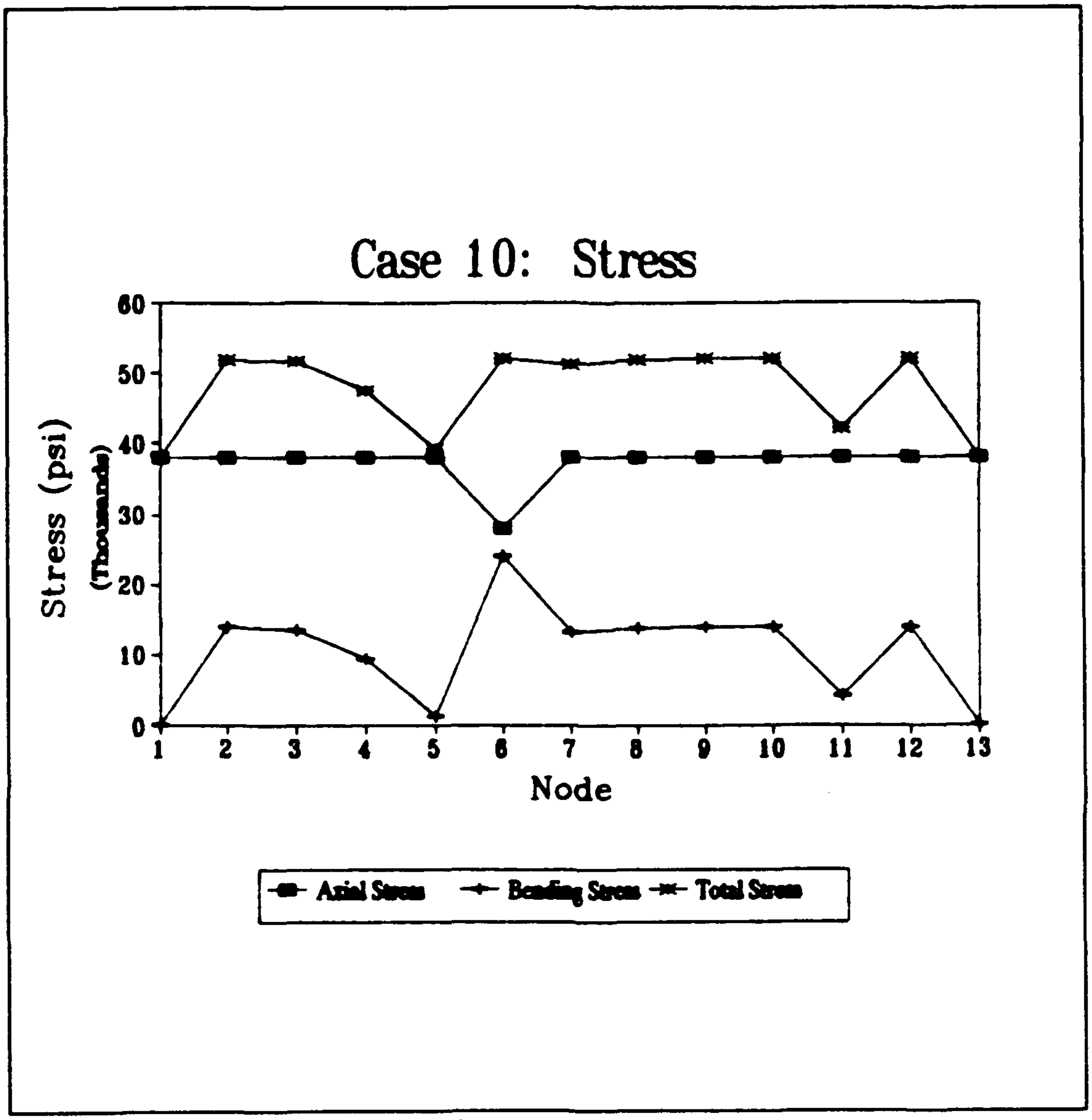

Figure 6.33: Case 10 - Stress

VII. CONCLUSIONS

The conclusions of this study are:

- The objective function appears to have local optimum points, because the initial shape of the arch affected the output from ADS. This required a trial-and-error approach to find the true optimal design for each case.

- In the optimum designs, the base dimensions were reduced to the maximum extent possible, and the arch took a rounded shape to decrease bending moments and increase axial loading. One exception to this was the case of the fixed-free arch, which was unable to attain axial loading and took the shape of a straight, tapered beam. The only other exception was Case (4), the hinged-hinged arch with a concentrated moment.

- The smooth, curved shapes that most of the arches assumed confirmed that modelling an arch as a series of straight elements was a valid approach as long as the number of elements was relatively high (greater than eight).

- When using the combination of strategy and optimizer (S.L.P. and M.M.F.D.) that this investigation employed, it is critical that the initial design be feasible. Since the initial shape in all cases was a straight beam, the initial base dimensions had to be large enough to allow the beam to be able to withstand the bending stresses that would dominate in such a structure. If the initial design was not feasible, the resulting output from ADS had many violated constraints.

- The most inefficient method of spanning the distance from point A to point B was the cantilever beam (Case (7)).

- Case Studies (5) through (8) confirmed the conclusions of references 5 and 6 that, for a given loading, the more statically indeterminate the structure, the greater the efficiency. However, Case (1) contradicted this conclusion since it is less statically indeterminate than Case (8) but has a slightly smaller volume.

Suggested areas for future research are:

- Optimization of an arch with a more complex cross-section (i.e. thin-walled tubes, WF's, etc.)
- Optimization of arches with geometric constraints which prevent global buckling and local crippling.

APPENDIX A: JUSTIFICATION FOR OMITTING SHEAR STRESSES

(The following appendix is taken from ref. 5)

The shear stress distribution through a beam of rectangular cross-section has a parabolic distribution along the depth of the member. The maximum shear stress, located at the neutral axis of the beam, is (from ref. 10):

$$\tau_{max} = 1.5\frac{V}{A} \tag{B.1}$$

where:

τ_{max} = *maximum shear stress*

V = *shear force*

A = *cross-sectional area*

The maximum normal stress due to bending is given by:

$$\sigma_n = \frac{Mc}{I} \tag{B.2}$$

where:

σ_n = *maximum normal stress*

M = *bending moment*

c = *distance from N.A. to extreme fiber*

$I = \frac{bh^3}{12}$ = *moment of inertia*

Redefining the normal stress in terms of the cross-sectional dimensions yields:

$$\sigma_n = M(h/2)/(bh^3/12)$$
$$or: \qquad (B.3)$$
$$\sigma_n = 6M/hA$$

The ratio of the maximum shear stress to the maximum normal stress due to bending is denoted by r and given by:

$$r = \frac{\tau_{max}}{\sigma_n} \qquad (B.4)$$

Substituting equations (B.1) and (B.3) into equation (B.4) yields:

$$r = (1.5V/A)/(6M/hA)$$
$$or: \qquad (B.5)$$
$$r = \frac{Vh}{4M}$$

For the cases investigated in this study, the maximum value r can attain is when the loading is that of a uniformly distributed load, p_y. In this case:

$$V = p_y L$$
$$and: \qquad (B.6)$$
$$M = p_y L^2/2$$

which upon substitution into equation (B.5) yields:

$$r = (p_y L)h/4(p_y L^2/2)$$

$$or: \qquad r = \frac{h}{2L} \tag{B.7}$$

The use of the beam equation requires that length of the beam to be at a minimum ten times the height, or in other words:

$$L \geq 10h \tag{B.8}$$

To maximize the value of r, let L equal 10h, the minimum allowable length. Substituting this value of L into Equation (B.7) yields:

$$r \leq h/2(10h)$$

$$or: \qquad r \leq 1/20 \tag{B.9}$$

Therefore, the maximum shear stress accounts for less than 5% of the bending stress developed in the structure. Five percent is high considering this analysis over-assumed the value of the shear stress by assigning the maximum shear stress to the entire cross-section of the beam. Moreover, at the outermost fibers where the normal stress is a maximum, the shear stress is zero. Therefore, under the circumstances of this study, the addition of shear stresses was deemed to be unwarranted.

FILE: ARCHOPT FORTRAN A

```
      PROGRAM ARCHOPT
C*****************************************************************
C*                                                               *
C*                ARCH OPTIMIZATION ANALYSIS CODE                *
C*                                                               *
C*****************************************************************
C
C  ALPHA....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO X-AXIS)
C  BAVE.....THE AVERAGE BASE DIMENSION ACROSS AN ELEMENT
C  BASE.....ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS
C  BASEL....ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS LOWER
C           SIDE CONSTRAINT
C  BASEU....ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS UPPER
C           SIDE CONSTRAINT
C  BETA ....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO Y-AXIS)
C  B_1......BOUNDARY TERMS APPLIED AT END "1"
C  B_2......BOUNDARY TERMS APPLIED AT END "2"
C  C1,..,C5.CONSTANTS RELATED TO ELEMENT STIFFNESS COEFFICIENTS
C  CLAN.....CONCENTRATED LOAD APPLICATION NODE (THE NODE FX,FY,FM ARE
C           APPLIED
C  DOF......DEGREE OF FREEDOMS (UNKNOWN DISPLACEMENTS & SLOPES)
C  DV1BG....DESIGN VARIABLE #1 (BASE DIMENSION) INITIAL ESTIMATE
C  DV1LO....DESIGN VARIABLE #1 (BASE DIMENSION) LOWER SIDE CONSTRAINT
C  DV1UP....DESIGN VARIABLE #1 (BASE DIMENSION) UPPER SIDE CONSTRAINT
C  DV2BG....DESIGN VARIABLE #2 (SLOPE) INITIAL ESTIMATE
C  DV2LO....DESIGN VARIABLE #2 (SLOPE) LOWER CONSTRAINT
C  DV2UP....DESIGN VARIABLE #2 (SLOPE) UPPER CONSTRAINT
C  EK.......6X6 ELEMENT STIFFNESS MATRIX IN LOCAL X,Y COORDINATES
C  EKPR.....6X6 ELEMENT STIFFNESS MATRIX IN ELEMENT LOCAL COORDINATES
C  ELEN.....LENGTH OF ELEMENT
C  F........FORCE VECTOR OF SYSTEM
C  FA.......CONSTANT DISTRIBUTED LOAD OUTWARD FROM END TO END
C  FM.......CONCENTRATED MOMENT AT CLAN
C  FX.......CONCENTRATED LOAD IN X DIRECTION AT CLAN
C  FY.......CONCENTRATED LOAD IN Y DIRECTION AT CLAN
C  G........THE ARRAY OF CONSTRAINT FUNCTIONS
C  GAMMA....6X6 ELEMENT TRANSFORMATION MATRIX
C  GK.......(NDOF)X(NDOF) GLOBAL STIFFNESS MATRIX
C  HGT......CONSTANT DEPTH OF CROSS-SECTION
C  INFO.....ADS PARAMETER USED TO SIGNAL THAT THE OPT IS COMPLETE
C  IPRINT...ADS PARAMETER USED SELECT THE DATA OUTPUT FORMAT
C  ITERATE..THE NUMBER OF TIMES ADS IS TO BE RELOADED WITH THE
C           PRECEEDING DATA
C  IWK......ADS INTERNAL WORK SPACE ARRAY
C  NCON.....NUMBER OF DESIGN CONSTRAINTS
C  NDOF.....NUMBER OF DEGREES OF FREEDOM
C  NDV......NUMBER OF DESIGN VARIABLES
C  NEL......NUMBER OF ELEMENTS
C  NRIWK....ADS INTERNAL WORK SPACE ARRAY DIMENSION
C  NRWK.....ADS INTERNAL WORK SPACE ARRAY DIMENSION
C  NSNP.....NUMBER OF SYSTEM NODAL POINTS
C  OBJ......THE OBJECTIVE FUNCTION OF THE OPTIMIZATION
C  OPTDCS...OPTIMIZATION DECISION TO OPTIMIZE THE PROBLEM OR NOT
C  P1...P5..PARAMETER DIMENSION CORRESPONDING TO THE NEL, NSNP, NCON,
C           NDOF, AND NDV RESPECTIVELY
C  PRCSN....THE PRECISION DESIRED TO SOLVE THE FEM SYSTEM OF EQUATIONS
C  SIGMA_B..THE ELEMENTAL NORMAL STRESS DUE TO BENDING
C  SIGMA_N..THE ELEMENTAL NORMAL STRESS DUE TO AXIAL FORCES
C  SIGMA_T..THE MAXIMUM TOTAL STRESS IN EACH ELEMENT
C  SX.......GLOBAL HORIZONTAL COORDINATE
C  SY.......GLOBAL VERTICAL COORDINATE
C  U........THE "DISPLACEMENT" VECTOR OF THE SYSTEM OF LINEAR EQUATIONS
C  VLB......ADS ARRAY CONTAINING UPPER SIDE CONSTRAINTS
C  VUB......ADS ARRAY CONTAINING LOWER SIDE CONSTRAINTS
C  WK.......ADS INTERNAL WORK AREA
C  X........ADS ARRAY CONTAINING THE VALUES OF THE DESIGN VARIABLES
C  YIELD....YIELD STRENGTH OF THE ARCH MATERIAL
C  YOUNG....YOUNG'S MODULUS OF THE ARCH MATERIAL
C*****************************************************************
C234567 ....DECLARE THE VARIABLES......................................
      INCLUDE 'ARCHCOM FORTRAN'
C
```

```
C        ....read the input parameters.................................
      OPEN(8, FILE='ARCHIN', STATUS='OLD')
C
      READ(8,*) L,H,HGT
      READ(8,*) YOUNG,YIELD
      READ(8,*) NEL,ISTRAT,IOPT,IONED,IPRINT,IGRAD
      READ(8,*) DV1BG,DV1LO,DV1UP
      READ(8,*) DV2BG,DV2LO,DV2UP
      READ(8,*) CLAN,FX,FY
      RE D(8,*) FM,FA,OPTDCS
      READ(8,*) ITERATE,PRCSN,BX1,BY1,BM1
      READ(8,*) BX2,BY2,BM2
      READ(8,*) LABEL
C
C        ....define constants..........................................
      NSNP = NEL + 1
      NDOF = 3*NSNP
      NCON = NSNP
      NDV = NSNP + (NEL - 1)
C
C
C        ....OPTIMIZE THE PROBLEM......................................
      CALL OPTIMIZATION_TOOL
C
C        ....COMPILE AND FORMAT THE OUTPUT.............................
      CALL ARCH_OUTPUT
C
      END
C**********************************************************************
C
      SUBROUTINE OPTIMIZATION_TOOL
C ====================================================================
C THIS SUBROUTINE DIRECTS THE PROGRAM FLOW OPTIMIZATION DECISION
C I.E., OPTIMIZE THE PROBLEM OR NOT.  IT ALSO SERVES TO SET UP &
C EXECUTE THE ADS OPTIMIZATION SOFTWARE.
C ====================================================================
C        ....declare the variables.....................................
      INCLUDE 'ARCHCOM FORTRAN'
      INTEGER I
      DO 100 I=1,NSNP
             BASE(I) = DV1BG
             BASEL(I) = DV1LO
             BASEU(I) = DV1UP
 100  CONTINUE
      DO 150 I=1,NEL-1
             SLP(I) = DV2BG
             SLPL(I) = DV2LO
             SLPU(I) = DV2UP
 150  CONTINUE
      SLP(NEL)=DV2BG
      SLPL(NEL)=DV2LO
      SLPU(NEL)=DV2UP
C
C        ....COMBINE BASE AND SLP ARRAYS INTO DESIGN ARRAY.........
      DO 200 I=1,NSNP
            X(I) = BASE(I)
            VLB(I) = BASEL(I)
            VUB(I) = BASEU(I)
 200  CONTINUE
      DO 250 J=NSNP+1,NDV
            X(J) = SLP(J-NSNP)
            VLB(J)=SLPL(J-NSNP)
            VUB(J)=SLPU(J-NSNP)
 250  CONTINUE
C
C ....MAKE OPTIMIZATION DECISION..............................
      IF (OPTDCS .NE. 1) THEN
        CALL EVAL
        RETURN
      ENDIF
C
C        ....DEFINE THE SIZE OF THE WORK ARRAYS FOR ADS................
```

```
      NRA=65
      NCOLA=97
      NRWK = 40000
      NRIWK = 2000
C        ....ready to optimize.......................................
      DO 280 I=1,NSNP
           IDG(I)=0
 280  CONTINUE
      INFO =-2
      CALL ADS (INFO,ISTRAT,IOPT,IONED,IPRINT,IGRAD,NDV,NCON,X,VLB,VUB,
     :         OBJ,G,IDG,NGT,IC,DF,A,NRA,NCOLA,WK,NRWK,IWK,NRIWK)
      IWK(2)=0

  300 CALL ADS (INFO,ISTRAT,IOPT,IONED,IPRINT,IGRAD,NDV,NCON,X,VLB,VUB,
     :         OBJ,G,IDG,NGT,IC,DF,A,NRA,NCOLA,WK,NRWK,IWK,NRIWK)
C
C        ....evaluate the objective function and constraints..........
      IF (INFO .NE. 0) THEN
           CALL EVAL
           GOTO 300
      ENDIF
      END
C*********************************************************************
C
      SUBROUTINE EVAL
C ===================================================================
C THIS SUBROUTINE IS USED TO EVALUATE THE OBJECTIVE FUNCTION,
C CONSTRAINT FUNCTIONS, AND SIDE CONSTRAINTS OF THE OPTIMIZATION
C PROBLEM.
C ===================================================================
C        ....declare the variables...................................
      INCLUDE 'ARCHCOM FORTRAN'
      INTEGER I,J,K
      REAL PI
      PARAMETER (PI=3.141593)
C
C        ....SEPARATE THE DESIGN ARRAY INTO BASE AND SLP ARRAYS
      DO 50 I=1,NSNP
            BASE(I) = X(I)
  50  CONTINUE
      DO 75 I=1,NDV-NSNP
            SLP(I) = X(I+NSNP)
  75  CONTINUE
      DX=L/NEL
      SY(1)=0.0
      DO 80 I=1,NEL-1
        SY(I+1)=DX*SLP(I) + SY(I)
 80   CONTINUE
      SLP(NEL)=(H - SY(NEL))/DX
C
C        ....calculate the objective function.........................
      OBJ = 0.0
C
      DO 100 I=1,NEL
         BAVE(I) = (BASE(I)+BASE(I+1))/2.0
         OBJ = OBJ + BAVE(I)*SQRT(1 + SLP(I)**2)
  100 CONTINUE
C        ....CALCULATE ALPHA(I),BETA(I), AND ELEN(I)...........
      DO 150 I=1,NEL
          ALPHA(I)=ATAN2(SLP(I)*DX,DX)
          BETA(I)=(PI/2) - ALPHA(I)
          ELEN(I)=SQRT(DX**2 + (SLP(I)*DX)**2)
 150  CONTINUE
C
C        ....determine the design constraints.........................
      CALL ARCH_STRESS
C
      DO 230 I=1,NSNP
           G(I) = (SIGMA_T(I)/YIELD - 1.0)
  230 CONTINUE
      END
C*********************************************************************
```

```
      SUBROUTINE ARCH_STRESS
C ===================================================================
C   THIS SUBROUTINE IS USED TO PERFORM THE FINITE ELEMENT ANALYSIS
C   OF THE STRESSES DEVELOPED IN AN ARCH OR BEAM FOR A GIVEN LOAD-
C   ING.
C ===================================================================
C         ....declare the variables.....................................
      INCLUDE 'ARCHCOM FORTRAN'
      INTEGER IPVT(99)
      REAL    F(P4)
      REAL*8 BK(P4,P4),BF(P4),BU(P4),FAC(9801),WORK(99)

C
C         ....form the element and system matrices......................
      CALL FORM
C
C       ....FORM THE FORCE VECTOR, F....................................
      CALL FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C
C       ....SET THE BOUNDARY CONDITIONS AND LOADS.......................
      CALL BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,BY2,BM2)
C
C       ....SOLVE THE SYSTEM OF EQUATIONS...............................
      IF (PRCSN .EQ. 2) THEN
C       ....CHANGE GK AND F ARRAYS TO DOUBLE PRECISION........
      CALL UPSCALE (NDOF,GK,F,BK,BF)
C       ....SOLVE THE SYSTEM OF EQUATIONS...............................
      CALL DL2ARG (NDOF,BK,P4,BF,1,BU,FAC,IPVT,WORK)
C       ....CHANGE BU ARRAY TO SINGLE PRECISION.........................
      CALL DOWNSCALE (NDOF,BU,U)
      ELSE
C       ....SOLVE THE SYSTEM OF EQUATIONS...............................
      CALL L2ARG (NDOF,GK,P4,F,1,U,FAC,IPVT,WORK)
      ENDIF
C
C         ....determine the stress distribution.........................
      CALL STRESS
C
C     RETURN
      END
C*******************************************************************
      SUBROUTINE FORM
C ===================================================================
C         This subroutine is used to construct the global stiffness mat-
C         RIX FOR THE ARCH PROBLEM.
C ===================================================================
C         ....declare the variables.....................................
      INCLUDE 'ARCHCOM FORTRAN'
      INTEGER IEL,I,J,K,II,JJ,KK,III,JJJ
      REAL    C1,C2,C3,C4,C5,CA,CB,EK(P1,6,6),GAMMA(6,6),EKGA(6,6),
     :        GAEKGA(6,6),BH,BH3
C
      DO 120 IEL=1,NEL
C         ....define the constants Cx...................................
      C1 = YOUNG/ELEN(IEL)
      C2 = (1.0/ELEN(IEL))**2.0
      C3 = (1.0)/(2.0*ELEN(IEL))
      C4 = (1.0)/3.0
      C5 = (1.0)/6.0
C
C         ....initialize the work arrays................................
        DO 100 I = 1,6
           DO 90 J= 1,6
                  EKPR(IEL,I,J) = 0.0
                  GAMMA(I,J) = 0.0
                  EKGA(I,J) = 0.0
                  GAEKGA(I,J) = 0.0
                  EK(IEL,I,J) = 0.0
   90            CONTINUE
  100        CONTINUE
C
C         ....calculate the area and inertia terms......................
```

```
      BH = BAVE(IEL)*HGT
      BH3 = BAVE(IEL)*(HGT**3.0)
C
C     ....determine the EKPR matrix.................................
      EKPR(IEL,1,1) =  C1*BH
      EKPR(IEL,1,4) = -C1*BH
      EKPR(IEL,2,2) =  C1*C2*BH3
      EKPR(IEL,2,3) =  C1*C3*BH3
      EKPR(IEL,2,5) = -C1*C2*BH3
      EKPR(IEL,2,6) =  C1*C3*BH3
      EKPR(IEL,3,2) =  C1*C3*BH3
      EKPR(IEL,3,3) =  C1*C4*BH3
      EKPR(IEL,3,5) = -C1*C3*BH3
      EKPR(IEL,3,6) =  C1*C5*BH3
      EKPR(IEL,4,1) = -C1*BH
      EKPR(IEL,4,4) =  C1*BH
      EKPR(IEL,5,2) = -C1*C2*BH3
      EKPR(IEL,5,3) = -C1*C3*BH3
      EKPR(IEL,5,5) =  C1*C2*BH3
      EKPR(IEL,5,6) = -C1*C3*BH3
      EKPR(IEL,6,2) =  C1*C3*BH3
      EKPR(IEL,6,3) =  C1*C5*BH3
      EKPR(IEL,6,5) = -C1*C3*BH3
      EKPR(IEL,6,6) =  C1*C4*BH3
C
C     ....determine the GAMMA matrix................................
         CA = COS(ALPHA(IEL))
         CB = COS(BETA(IEL))
      GAMMA(1,1) =  CA
      GAMMA(1,2) =  CB
      GAMMA(2,1) = -CB
      GAMMA(2,2) =  CA
      GAMMA(3,3) =  1.0
      GAMMA(4,4) =  CA
      GAMMA(4,5) =  CB
      GAMMA(5,4) = -CB
      GAMMA(5,5) =  CA
      GAMMA(6,6) =  1.0

C
C     ....determine the EKGA array..................................
         DO 220 I = 1,6
            DO 215 J = 1,6
               DO 210 K = 1,6
                  EKGA(I,J) = EKGA(I,J) + EKPR(IEL,I,K)*GAMMA(K,J)
  210          CONTINUE
  215       CONTINUE
  220    CONTINUE
C
C     ....determine the GAEKGA array................................
         DO 240 I = 1,6
            DO 235 J = 1,6
               DO 230 K = 1,6
                  GAEKGA(I,J) = GAEKGA(I,J)+GAMMA(K,I)*EKGA(K,J)
  230          CONTINUE
  235       CONTINUE
  240    CONTINUE
C
C     ....copy the GAEKGA array into the EK array...................
         DO 260 I = 1,6
            DO 250 J = 1,6
               EK(IEL,I,J) = GAEKGA(I,J)
  250       CONTINUE
  260    CONTINUE
  120 CONTINUE
C
C     ....initialize the GK array...................................
      DO 150 I = 1, NDOF
         DO 140 J = 1, NDOF
            GK(I,J) = 0.0
  140    CONTINUE
  150 CONTINUE
```

```
C
C       ....construct the GK matrix.................................
        DO 300 IEL = 1, NEL
                II = 3*(IEL-1)
           DO 290 J = 1, 6
                 JJ = II + J
                 DO 280 K = 1, 6
                       KK = II + K
                    GK(JJ,KK) = GK(JJ,KK)+EK(IEL,J,K)
  280            CONTINUE
  290      CONTINUE
  300   CONTINUE
C
C       RETURN
        END
C*********************************************************************
      SUBROUTINE FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C ===================================================================
C       This subroutine is used to construct the force vector for the
C       FEM PROBLEM SPECIFIED.
C ===================================================================
C       ....DECLARE THE VARIABLES......................................
      INTEGER NEL,NDOF,I,I1,I2,I3,P1,P4
C
      PARAMETER(P1=32,P4=99)
C
      REAL ELEN(P1),ALPHA(P1),BETA(P1),FA,F(P4)
C
C       ...FORM THE F-VECTOR.........................................
      F(1) = (ELEN(1)/2.0) * (-COS(BETA(1)))
      F(2) = (ELEN(1)/2.0) * (COS(ALPHA(1)))
      F(3) = 0.0
C
      DO 100 I=2,NEL
           I1 = (I-1)*3 + 1
           I2 = (i-1)*3 + 2
           I3 = (i-1)*3 + 3
C
      F(I1) = (ELEN(I)/2.0)*(-COS(BETA(I)))
     :      +(ELEN(I-1)/2.0)*(-COS(BETA(I-1)))
      F(I2) = (ELEN(I)/2.0)*(COS(ALPHA(I)))
     :      +(ELEN(I-1)/2.0)*(COS(ALPHA(I-1)))
      F(I3) = 0.0
 100  CONTINUE
C
      F(NDOF-2) = (ELEN(NEL)/2.0)*(-COS(BETA(NEL)))
      F(NDOF-1) = (ELEN(NEL)/2.0)*(COS(ALPHA(NEL)))
      F(NDOF)   = 0.0
C
C       ....SCALE THE F-VECTOR BY FA.................................
      DO 200 I=1,NDOF
        F(I) = FA*F(I)
 200  CONTINUE
C
C        RETURN
         END
C**********************************************************************
      SUBROUTINE BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,
     :             BY2,BM2)
C ===================================================================
C       This subroutine is used to impose the boundary conditions upon
C       THE GLOBAL STIFFNESS MATRIX AND FORCE VECTOR.
C ====================================================================
C       ....declare the variables....................................
      INTEGER NDOF,BX1,BY1,BM1,BX2,BY2,BM2,CLAN,I,N,I1,I2,I3,P4
      PARAMETER(P4=99)
      REAL    GK(P4,P4),FX,FY,FM,F(P4)
C
C       ....invoke the essential boundary conditions..................
      IF (BX1 .EQ. 1) THEN
      CALL IMPOSEBC (NDOF,GK,1,F)
      ENDIF
```

```
C
      IF (BY1 .EQ. 1) THEN
          CALL IMPOSEBC (NDOF,GK,2,F)
      ENDIF
C
      IF (BM1 .EQ. 1) THEN
          CALL IMPOSEBC (NDOF,GK,3,F)
      ENDIF
C
      IF (BX2 .EQ. 1) THEN
          N=NDOF-2
          CALL IMPOSEBC (NDOF,GK,N,F)
      ENDIF
C
      IF (BY2 .EQ. 1) THEN
          N=NDOF-1
          CALL IMPOSEBC (NDOF,GK,N,F)
      ENDIF
C
      IF (BM2 .EQ. 1) THEN
          CALL IMPOSEBC (NDOF,GK,NDOF,F)
      ENDIF
C
C ....ADD THE CONCENTRATED LOAD TO THE FORCE VECTOR...........
        I1=(CLAN-1)*3+1
        I2=(CLAN-1)*3+2
        I3=(CLAN-1)*3+3
C
        F(I1)=F(I1)+FX
        F(I2)=F(I2)+FY
        F(I3)=F(I3)+FM
C
C     RETURN
      END
C*********************************************************************
      SUBROUTINE IMPOSEBC (NDOF,GK,N,F)
C ===================================================================
C       This subroutine is used to do the redundant leg work of impos-
C       ING THE BOUNDARY CONDITIONS.
C ===================================================================
C       ....DECLARE THE VARIABLES...................................
      INTEGER NDOF,N,I,P4
      PARAMETER(P4=99)
      REAL GK(P4,P4),F(P4)
C
C     ....IMPOSE THE BOUNDARY CONDITION ON THE GK AND F ARRAYS......
      DO 100 I=1,NDOF
        GK(N,I) = 0.0
 100  CONTINUE
      GK(N,N) = 1.0
      F(N) = 0.0
C
C     RETURN
      END
C*********************************************************************
      SUBROUTINE UPSCALE(NDOF,GK,F,BK,BF)
C ===================================================================
C       This subroutine is used to change the stiffness matrix & force
C       VECTOR FROM SINGLE PRECISION TO DOUBLE PRECISION IN ORDER TO
C       SOLVE THE LINEAR SYSTEM OF EQUATIONS IN DOUBLE PRECISION.
C ===================================================================
C     ....DECLARE THE VARIABLES...................................
      INTEGER NDOF,I,J,P4
      PARAMETER (P4=99)
      REAL GK(P4,P4),F(P4)
      REAL*8 BK(P4,P4),BF(P4)
C
C     ....GENERATE THE DOUBLE PRECISION COMPLIMENTS OF GK AND F......
      DO 110 I=1,NDOF
      DO 100 J=1,NDOF
      BK(I,J) = GK(I,J)
 100  CONTINUE
```

```
      BF(I) = F(I)
 110  CONTINUE
C
C     RETURN
      END
C*****************************************************************
      SUBROUTINE DOWNSCALE(NDOF,BU,U)
C ===============================================================
C        This subroutine is used to do down scale the double precision
C        SOLUTION OF THE LINEAR SYSTEM OF EQUATIONS BACK TO SINGLE PRE-
C        CISION. ADS COULD HAVE PROBLEMS WITH DOUBLE PRECISION NUMBERS!
C ===============================================================
C        ....declare the variables..................................
      INTEGER NDOF,I,P4
      PARAMETER (P4=99)
      REAL    U(P4)
      REAL*8  BU(P4)
C
C        ....GENERATE THE SINGLE PRECISION COMPLIMENT OF BU......
      DO 100 I=1,NDOF
           U(i) = BU(i)
 100  CONTINUE
C
C     RETURN
      END
C****************************************************************
      SUBROUTINE STRESS
C  ==============================================================
C     THIS SUBROUTINE COMPUTES THE STRESS AT EACH NODAL POINT.
C  ==============================================================
C        ....declarations...........................................
      INCLUDE 'ARCHCOM FORTRAN'
      INTEGER    I1,I2,I3,I4,I5,I6,I,J,K
      REAL       CA1,CB1,K1,K2,FPR(P4,6),UPR(6),NORM1,NORM2,
     :              BEND1,BEND2
C
C        ....determine local forces from stiffness and displacement....
      DO 100 I=1, NEL
           I1=(I-1)*3+1
           I2=(i-1)*3+2
           I3=(i-1)*3+3
           I4=(i)*3+1
           I5=(i)*3+2
           I6=(i)*3+3
C
           CB1= COS(BETA(I))
           CA1= COS(ALPHA(I))
C
           UPR(1)=  U(I1)*CA1 +  U(I2)*CB1
           UPR(2)= -U(I1)*CB1 +  U(I2)*CA1
           UPR(3)=  U(I3)
           UPR(4)=  U(I4)*CA1 +  U(I5)*CB1
           UPR(5)= -U(I4)*CB1 +  U(I5)*CA1
           UPR(6)=  U(I6)
C
              DO 250 JJ=1,6
                 FPR(I,JJ)= 0.0
  250         CONTINUE
C
              DO 300 J=1,6
                 DO 350 K=1,6
                    FPR(I,J)= FPR(I,J) + EKPR(I,J,K)*UPR(K)
  350            CONTINUE
  300         CONTINUE
  100    CONTINUE
C        ....determine the bending and normal stresses................
         SIGMA_N(1) = ABS(FPR(1,1)*(1.0/(BASE(1)*HGT)))
         SIGMA_B(1) = ABS(FPR(1,3)*(6.0/(BASE(1)*(HGT**2.0))))
         SIGMA_T(1) = SIGMA_B(1) + SIGMA_N(1)
         DO 400 i=2,NEL
               K1 = 1.0/(BASE(I)*HGT)
               K2 = 6.0/(BASE(I)*(HGT**2.0))
```

```
            NORM1 = ABS(FPR(i,1)*K1)
            NORM2 = ABS(FPR(i-1,4)*K1)

            BEND1 = ABS(FPR(i,3)*K2)
            BEND2 = ABS(FPR(i-1,6)*K2)

            SIGMA_N(i) = (NORM1+NORM2)/2.0
            SIGMA_B(i) = (BEND1+BEND2)/2.0

            SIGMA_T(i) = SIGMA_B(i) + SIGMA_N(i)

  400    CONTINUE

         SIGMA_N(NSNP) = ABS(FPR(NEL,4)*(1.0/(BASE(NSNP)*HGT)))
         SIGMA_B(NSNP) = ABS(FPR(NEL,6)*
     :                   (6.0/(BASE(NSNP)*(HGT**2.0))))
         SIGMA_T(NSNP) = SIGMA_B(NSNP) + SIGMA_N(NSNP)
C
C     RETURN
      END
C*********************************************************************
      SUBROUTINE ARCH_OUTPUT
C ===================================================================
C       THIS SUBROUTINE FORMATS THE FINAL RESULTS AND OUTPUT OF THE
C       OPTIMIZATION PROBLEM AND STORES IT IN A FILE NAMED ARCH_OUT
C ===================================================================
C     ....DECLARE VARIABLES.........................................
      INCLUDE 'ARCHCOM FORTRAN'
      REAL     VOL,VOLUME
C
C     ....OPEN OUTPUT FILE AND WRITE HEADER.........................
      OPEN(9, FILE='ARCHOUT', STATUS='OLD')
C
      WRITE(9,100) LABEL
      WRITE(9,100) ' OPTIMIZATION SOLUTION'
      WRITE(9,105) ' ----------------------------------------------
     :----------------------'
 100  FORMAT(/5X,A)
 105  FORMAT(5X,A)
C
C       ....SECTION "A"...............................................
      WRITE(9,100) ' A) PROBLEM PARAMETERS:'
      WRITE(9,110) ' HORIZ. SPAN:', L, ' YOUNGS MODULUS:',YOUNG
      WRITE(9,110) ' VERT. SPAN :', H, ' YIELD STRENGTH:',YIELD
      WRITE(9,115) ' NO OF DESIGN VAR:', NDV, ' NO OF ELEMENTS:',NEL
 110  FORMAT(8X,A,F12.3,T38,A,F12.1)
 115  FORMAT(8X,A,I7,T38,A,I10)
C
C       ....SECTION "B"...............................................
      WRITE(9,100) ' B) DERIVED CONSTANTS:'
      WRITE(9,120) ' NO OF SYSTEM NODAL POINTS...',NSNP
      WRITE(9,120) ' NO OF DEGREES OF FREEDOM....',NDOF
      WRITE(9,125) ' HORIZ. LENGTH PER ELEMENT...',DX
 120  FORMAT(8X,A,I6)
 125  FORMAT(8X,A,F12.4)
C
C        ....section "C"..............................................
      WRITE(9,100) ' C) STRUCTURE LOADING:'
      WRITE(9,125) 'FX..........................',FX
      WRITE(9,125) 'FY..........................',FY
      WRITE(9,125) 'FM......................... .',FM
      WRITE(9,125) 'FA..........................',FA
      WRITE(9,120) 'CONCENTRATED LOAD AT NODE...',CLAN
C
C        ....section "D"..............................................
      SX(1)=0.0
      SY(1)=0.0
      DO 150 I=1,NEL
         SX(I+1)=I*DX
         SY(I+1)=DX*SLP(I) + SY(I)
 150  CONTINUE
```

```
      WRITE(9,100) ' D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:'
      WRITE(9,210) 'NODE','X-COORD','Y-COORD','BASE','AREA'
C
 210  FORMAT(8X,A,T21,A,T36,A,T49,A,T62,A)
 220  FORMAT(8X,I4,T17,F10.5,T32,F10.5,T48,F8.5,T60,F8.5)
        VOLUME = 0.0
C
      DO 300 I=1,NSNP
          AREA = BASE(I)*HGT
          WRITE(9,220) I,SX(I),SY(I),BASE(I),AREA
 300  CONTINUE
C
C       ....section "E"...........................................
      VOL = HGT*DX*OBJ
      WRITE(9,100) ' E) OBJECTIVE FUNCTION:'
C
      WRITE(9,310) ' TOTAL STRUCTURE VOLUME:',VOL
 310  FORMAT(/12X,A,F12.6/)
C
      WRITE(9,330) 'NODE','NORMAL STRESS','BENDING STRESS','TOTAL'
      DO 320 I=1,NSNP
          WRITE(9,340) i,SIGMA_N(i),SIGMA_B(i),SIGMA_T(i)
  320 CONTINUE
  330 FORMAT(8X,A,T18,A,T35,A,T57,A)
  340 FORMAT(8X,I4,T15,F14.1,T32,F14.1,T49,F14.1)
C
C       ....section "F"...........................................
      WRITE(9,100) ' F) BOUNDARY CONDITIONS:'
      WRITE(9,410) 'NODE','X-DISPL','Y-DISPL','SLOPE'
      WRITE(9,430) 1,BX1,BY1,BM1
      WRITE(9,430) NEL+1,BX2,BY2,BM2
C
C       ....SECTION "G"...........................................
      WRITE(9,100) ' G) SOLUTION VECTOR:'
      WRITE(9,410) 'NODE','X-DISPL','Y-DISPL','SLOPE'
      DO 400 I=1,NSNP
       I1=(I-1)*3+1
       I2=(I-1)*3+2
       I3=(I-1)*3+3
      WRITE(9,420) I,U(I1),U(I2),U(I3)
 400  CONTINUE
 410  FORMAT(T9,A,T17,A,T31,A,T46,A)
 420  FORMAT(7X,I5,3E14.6)
 430  FORMAT(7X,I5,T20,I4,T34,I4,T48,I4)
C
C     RETURN
      END
```

OPTIMIZATION NO.1

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX..........................	0.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	-100.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15000	0.22500
2	2.66667	4.30175	0.15000	0.22500
3	5.33333	8.23110	0.15084	0.22627
4	8.00000	11.74150	0.15022	0.22533
5	10.66666	14.87895	0.15092	0.22633
6	13.33333	17.72124	0.15014	0.22522
7	16.00000	20.31323	0.15000	0.22500
8	18.66666	22.73409	0.15000	0.22500
9	21.33333	24.91997	0.15000	0.22500
10	23.99998	26.86490	0.15000	0.22500
11	26.66666	28.66069	0.15000	0.22500
12	29.33331	30.59526	0.15000	0.22500
13	31.99998	31.99998	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 10.324365

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	36039.3	0.1	36039.5
2	36031.7	15159.3	51191.0
3	35823.3	16175.0	51998.3
4	35975.3	15313.8	51289.2
5	35809.8	15699.2	51509.1
6	35994.7	16009.5	52004.3
7	36033.5	15762.4	51796.0
8	36044.7	9766.7	45811.4
9	36049.8	7516.0	43565.8
10	36041.4	12457.1	48498.6
11	36016.8	16026.7	52043.5
12	36039.7	15969.3	52009.0
13	36081.4	0.0	36081.4

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.131531E-01
2	0.509330E-01	-0.387271E-01	-0.114481E-01
3	0.863026E-01	-0.696029E-01	-0.814113E-02
4	0.106222E+00	-0.913596E-01	-0.505612E-02
5	0.114465E+00	-0.104831E+00	-0.221819E-02
6	0.113685E+00	-0.110495E+00	0.528031E-03
7	0.105703E+00	-0.108688E+00	0.315360E-02

FILE: FILE CASE1 A

8	0.921987E-01	-0.100249E+00	0.519679E-02
9	0.761242E-01	-0.871751E-01	0.652113E-02
10	0.589296E-01	-0.703301E-01	0.798611E-02
11	0.396341E-01	-0.485910E-01	0.100210E-01
12	0.162884E-01	-0.231430E-01	0.100252E-01
13	0.000000E+00	0.000000E+00	0.895565E-02

FILE: FILE CASE1A A

OPTIMIZATION NO.1A

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS... 13
NO OF DEGREES OF FREEDOM.... 39
HORIZ. LENGTH PER ELEMENT... 2.6667

C) STRUCTURE LOADING:

FX......................... 0.0000
FY......................... 0.0000
FM......................... 0.0000
FA......................... -100.0000
CONCENTRATED LOAD AT NODE... 7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.79198	1.18797
2	2.66667	2.66933	0.78408	1.17612
3	5.33333	5.33867	0.78335	1.17503
4	8.00000	8.00800	0.98644	1.47966
5	10.66666	10.67200	1.16866	1.75299
6	13.33333	13.33852	1.27656	1.91484
7	16.00000	16.00357	1.30861	1.96291
8	18.66666	18.66756	1.27457	1.91186
9	21.33333	21.33154	1.16462	1.74693
10	23.99998	23.99666	0.98731	1.48096
11	26.66666	26.66280	0.78324	1.17486
12	29.33331	29.32883	0.78168	1.17252
13	31.99998	31.99998	0.78982	1.18473

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 68.386749

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	91.5	0.0	91.5
2	92.6	26676.4	26769.0
3	92.2	48556.6	48648.8
4	72.9	52068.1	52140.9
5	61.6	52094.5	52156.1
6	56.0	52183.3	52239.4
7	54.3	52353.9	52408.3
8	56.5	52263.8	52320.3
9	62.2	52273.6	52335.8
10	73.0	52014.1	52087.1
11	92.3	48564.5	48656.7
12	91.5	26770.9	26862.4
13	89.5	0.2	89.8

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.432361E-01
2	0.113423E+00	-0.113326E+00	-0.410106E-01
3	0.215283E+00	-0.215100E+00	-0.347033E-01
4	0.297166E+00	-0.296917E+00	-0.262325E-01
5	0.355750E+00	-0.355571E+00	-0.175074E-01
6	0.390952E+00	-0.390785E+00	-0.876822E-02
7	0.402699E+00	-0.402549E+00	-0.976327E-05

FILE: FILE CASE1A A

8	0.390992E+00	-0.390840E+00	0.875361E-02
9	0.355829E+00	-0.355653E+00	0.175100E-01
10	0.297182E+00	-0.296983E+00	0.262491E-01
11	0.215351E+00	-0.215151E+00	0.347106E-01
12	0.113571E+00	-0.113364E+00	0.410251E-01
13	0.000000E+00	0.000000E+00	0.432590E-01

FILE: FILE CASE2 A

OPTIMIZATION NO.2

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	0.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15000	0.22500
2	2.66667	3.00301	0.15000	0.22500
3	5.33333	6.30157	0.15000	0.22500
4	8.00000	9.60067	0.15000	0.22500
5	10.66666	12.91733	0.15000	0.22500
6	13.33333	16.27870	0.15000	0.22500
7	16.00000	19.31017	0.15000	0.22500
8	18.66666	21.33359	0.15000	0.22500
9	21.33333	23.39781	0.15000	0.22500
10	23.99998	25.46082	0.15000	0.22500
11	26.66666	27.54510	0.15000	0.22500
12	29.33331	29.74944	0.15000	0.22500
13	31.99998	31.99998	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 10.239457

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	31082.2	0.3	31082.6
2	31095.7	20958.1	52053.9
3	31109.7	18671.2	49780.9
4	31109.4	16336.5	47445.9
5	31107.6	12623.3	43730.9
6	31097.4	5386.4	36483.7
7	27975.7	24094.0	52069.8
8	24863.7	27275.3	52139.0
9	24863.7	27252.7	52116.4
10	24863.8	27320.7	52184.5
11	24857.5	25720.0	50577.5
12	24846.4	14678.1	39524.5
13	24841.8	0.0	24841.8

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.169057E-01
2	0.461328E-01	-0.465305E-01	-0.150352E-01
3	0.866829E-01	-0.849686E-01	-0.112998E-01
4	0.115632E+00	-0.114025E+00	-0.799962E-02
5	0.134663E+00	-0.134988E+00	-0.526077E-02
6	0.146308E+00	-0.149906E+00	-0.354366E-02
7	0.151127E+00	-0.159717E+00	-0.898687E-03

FILE: FILE CASE2 A

8	0.146949E+00	-0.158800E+00	0.292259E-02
9	0.134488E+00	-0.147268E+00	0.700891E-02
10	0.113602E+00	-0.124838E+00	0.110977E-01
11	0.840616E-01	-0.915992E-01	0.150871E-01
12	0.448603E-01	-0.486744E-01	0.181930E-01
13	0.000000E+00	0.000000E+00	0.193311E-01

FILE: FILE CASE3 A

OPTIMIZATION NO.3

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX.........................	2000.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	0.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15030	0.22545
2	2.66667	3.57692	0.15037	0.22556
3	5.33333	7.11881	0.15045	0.22568
4	8.00000	11.13781	0.15079	0.22618
5	10.66666	14.64122	0.15017	0.22526
6	13.33333	18.30432	0.15000	0.22500
7	16.00000	21.47563	0.23559	0.35339
8	18.66666	23.44431	0.15046	0.22568
9	21.33333	25.24850	0.15008	0.22512
10	23.99998	26.98657	0.15068	0.22603
11	26.66666	28.49719	0.15040	0.22561
12	29.33331	30.15172	0.15009	0.22514
13	31.99998	31.99998	0.15012	0.22517

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 10.841561

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	32852.1	0.1	32852.2
2	32835.8	8291.1	41126.9
3	32805.9	19268.1	52074.0
4	32730.9	6463.6	39194.5
5	32878.3	7478.3	40356.5
6	32878.2	9143.1	42021.4
7	21034.9	31050.3	52085.2
8	33175.7	16387.2	49562.9
9	33301.1	2650.6	35951.7
10	33157.7	3655.2	36812.9
11	33220.8	15609.0	48829.8
12	33293.2	18737.4	52030.6
13	33272.5	0.1	33272.6

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.927029E-02
2	0.292592E-01	-0.279060E-01	-0.844808E-02
3	0.520934E-01	-0.511700E-01	-0.573258E-02
4	0.676226E-01	-0.677945E-01	-0.436322E-02
5	0.806200E-01	-0.837384E-01	-0.426672E-02
6	0.903615E-01	-0.969766E-01	-0.259323E-02
7	0.912367E-01	-0.102322E+00	0.155546E-02

FILE: FILE CASE3 A

8	0.815972E-01	-0.940706E-01	0.528782E-02
9	0.675751E-01	-0.797142E-01	0.665123E-02
10	0.529933E-01	-0.638010E-01	0.657931E-02
11	0.398219E-01	-0.474255E-01	0.739218E-02
12	0.227118E-01	-0.264506E-01	0.978718E-02
13	0.000000E+00	0.000000E+00	0.111381E-01

FILE: FILE CASE4 A

OPTIMIZATION NO.4

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS... 13
NO OF DEGREES OF FREEDOM.... 39
HORIZ. LENGTH PER ELEMENT... 2.6667

C) STRUCTURE LOADING:

FX......................... 0.0000
FY......................... 0.0000
FM......................... 6000.0000
FA......................... 0.0000
CONCENTRATED LOAD AT NODE... 7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15087	0.22631
2	2.66667	2.69333	0.15000	0.22500
3	5.33333	5.38662	0.15000	0.22500
4	8.00000	8.07996	0.15000	0.22500
5	10.66666	10.77329	0.15000	0.22500
6	13.33333	13.46662	0.15000	0.22500
7	16.00000	16.10661	0.15377	0.23066
8	18.66666	18.74661	0.15000	0.22500
9	21.33333	21.43994	0.15014	0.22521
10	23.99998	24.13327	0.15005	0.22507
11	26.66666	26.77327	0.15000	0.22500
12	29.33331	29.41327	0.15000	0.22500
13	31.99998	31.99998	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 10.207288

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	18.9	0.0	18.9
2	19.0	8935.8	8954.8
3	19.0	17871.6	17890.6
4	19.0	26806.6	26825.6
5	19.1	35742.1	35761.2
6	22.0	44677.8	44699.8
7	24.3	52024.6	52048.9
8	22.0	44303.3	44325.2
9	19.0	35335.9	35354.9
10	21.8	26425.4	26447.2
11	24.6	17591.1	17615.7
12	27.6	8749.7	8777.3
13	30.6	0.1	30.7

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.911602E-02
2	0.238804E-01	-0.236406E-01	-0.836558E-02
3	0.437104E-01	-0.432711E-01	-0.610774E-02
4	0.554325E-01	-0.548738E-01	-0.234471E-02
5	0.549926E-01	-0.544349E-01	0.292347E-02
6	0.383368E-01	-0.379406E-01	0.969686E-02
7	0.238511E-02	-0.162140E-02	0.177837E-01

FILE: FILE CASE4 A

8	-0.336480E-01	0.347801E-01	0.975857E-02
9	-0.505593E-01	0.515273E-01	0.305128E-02
10	-0.514326E-01	0.523953E-01	-0.215084E-02
11	-0.405827E-01	0.414402E-01	-0.582135E-02
12	-0.219884E-01	0.226625E-01	-0.801783E-02
13	0.000000E+00	0.000000E+00	-0.874020E-02

FILE: FILE CASE5 A

OPTIMIZATION NO.5

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX..........................	0.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	-100.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15000	0.22500
2	2.66667	10.66666	0.16746	0.25118
3	5.33333	16.43103	0.15000	0.22500
4	8.00000	20.18451	0.15000	0.22500
5	10.66666	22.93553	0.15000	0.22500
6	13.33333	25.15350	0.15006	0.22510
7	16.00000	26.92931	0.15000	0.22500
8	18.66666	28.34477	0.15000	0.22500
9	21.33333	29.47061	0.15000	0.22500
10	23.99998	30.29001	0.15000	0.22500
11	26.66666	30.90901	0.15000	0.22500
12	29.33331	31.43495	0.15000	0.22500
13	31.99998	31.99998	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 11.328694

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	13798.3	0.1	13798.5
2	12371.2	39641.7	52012.9
3	13857.5	38158.4	52015.9
4	13897.5	36095.7	49993.2
5	13901.5	38115.7	52017.2
6	13895.2	38108.5	52003.6
7	13903.2	38094.6	51997.8
8	13906.0	38087.9	51993.8
9	13909.4	37119.4	51028.8
10	13912.9	37831.8	51744.7
11	13924.4	35752.2	49676.6
12	13922.3	25555.1	39477.4
13	13910.7	1.5	13912.2

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	-0.595168E+00	0.000000E+00	-0.429371E-01
2	-0.174663E+00	-0.110052E+00	-0.327189E-01
3	-0.196828E-01	-0.184795E+00	-0.217266E-01
4	0.462421E-01	-0.234246E+00	-0.141289E-01
5	0.752630E-01	-0.264850E+00	-0.781040E-02
6	0.848358E-01	-0.278872E+00	-0.193522E-02
7	0.822190E-01	-0.277620E+00	0.349024E-02

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8	0.724252E-01	-0.262154E+00	0.860142E-02
9	0.587701E-01	-0.233261E+00	0.134390E-01
10	0.446241E-01	-0.191628E+00	0.180855E-01
11	0.307934E-01	-0.137662E+00	0.225620E-01
12	0.166607E-01	-0.725301E-01	0.262650E-01
13	0.000000E+00	0.000000E+00	0.278131E-01

FILE: FILE CASE6 A

OPTIMIZATION NO.6

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX.........................	0.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	-100.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15000	0.22500
2	2.66667	6.78236	0.30329	0.45494
3	5.33333	14.31105	0.24584	0.36876
4	8.00000	19.67329	0.15000	0.22500
5	10.66666	22.94662	0.15000	0.22500
6	13.33333	24.93712	0.15000	0.22500
7	16.00000	26.01016	0.15000	0.22500
8	18.66666	27.14098	0.15000	0.22500
9	21.33333	28.00891	0.15000	0.22500
10	23.99998	28.76320	0.15000	0.22500
11	26.66666	29.50047	0.15000	0.22500
12	29.33331	30.30164	0.15004	0.22506
13	31.99998	31.99998	0.43413	0.65119

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 14.430920

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	12224.7	0.1	12224.8
2	6057.4	45945.8	52003.1
3	7547.9	44469.5	52017.4
4	12709.8	19397.3	32107.1
5	13071.6	8753.1	21824.7
6	13083.4	10261.1	23344.6
7	12983.4	11714.5	24698.0
8	12980.5	16215.1	29195.6
9	12969.5	18953.1	31922.6
10	12975.1	13532.8	26507.9
11	12951.8	4652.1	17603.9
12	12164.7	39871.4	52036.1
13	3942.6	48101.8	52044.4

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0	1	0
13	1	1	1

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	-0.238121E+00	0.000000E+00	-0.323442E-01
2	-0.419812E-01	-0.792292E-01	-0.223869E-01
3	0.630188E-01	-0.118314E+00	-0.631173E-02
4	0.662482E-01	-0.122027E+00	0.299570E-02
5	0.522159E-01	-0.112946E+00	0.399436E-02
6	0.444548E-01	-0.104995E+00	0.258829E-02
7	0.407255E-01	-0.990568E-01	0.268114E-02

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8	0.355767E-01	-0.901286E-01	0.447894E-02
9	0.296101E-01	-0.757135E-01	0.667067E-02
10	0.226288E-01	-0.554353E-01	0.867125E-02
11	0.147434E-01	-0.314062E-01	0.921718E-02
12	0.702276E-02	-0.987708E-02	0.646195E-02
13	0.000000E+00	0.000000E+00	0.000000E+00

FILE: FILE CASE7 A

OPTIMIZATION NO.7

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS...	13
NO OF DEGREES OF FREEDOM....	39
HORIZ. LENGTH PER ELEMENT...	2.6667

C) STRUCTURE LOADING:

FX.........................	0.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	-100.0000
CONCENTRATED LOAD AT NODE...	7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	1.31120	3.93359
2	2.66667	2.66377	1.10192	3.30575
3	5.33333	5.25856	0.91330	2.73989
4	8.00000	7.97715	0.73940	2.21821
5	10.66666	11.05349	0.57427	1.72280
6	13.33333	14.02065	0.43146	1.29439
7	16.00000	16.77490	0.31308	0.93925
8	18.66666	19.50882	0.30000	0.90000
9	21.33333	22.17821	0.30000	0.90000
10	23.99998	24.47510	0.30000	0.90000
11	26.66666	27.00243	0.30000	0.90000
12	29.33331	29.40999	0.30000	0.90000
13	31.99998	31.99998	0.30000	0.90000

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 72.614471

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	0.5	52070.8	52071.3
2	8.9	52066.6	52075.5
3	16.1	52056.2	52072.2
4	61.1	51982.5	52043.6
5	123.9	51879.7	52003.6
6	107.4	51886.1	51993.4
7	92.3	51919.3	52011.6
8	79.6	37074.2	37153.8
9	60.3	23343.8	23404.1
10	28.9	13394.2	13423.0
11	9.9	5932.8	5942.7
12	7.7	1530.9	1538.6
13	0.0	5.7	5.7

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	1
13	0	0	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.597678E-02	-0.598316E-02	-0.436127E-02
3	0.230543E-01	-0.235304E-01	-0.866592E-02
4	0.528078E-01	-0.527179E-01	-0.130684E-01
5	0.100533E+00	-0.941091E-01	-0.177673E-01
6	0.160387E+00	-0.147924E+00	-0.223668E-01
7	0.228394E+00	-0.213783E+00	-0.267883E-01

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8	0.307128E+00	-0.290595E+00	-0.305781E-01
9	0.392384E+00	-0.375776E+00	-0.331111E-01
10	0.470241E+00	-0.466157E+00	-0.345477E-01
11	0.558680E+00	-0.559472E+00	-0.353367E-01
12	0.644186E+00	-0.654177E+00	-0.356349E-01
13	0.736590E+00	-0.749316E+00	-0.356983E-01

FILE: FILE CASE8 A

OPTIMIZATION NO.8

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	32.000	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS... 13
NO OF DEGREES OF FREEDOM.... 39
HORIZ. LENGTH PER ELEMENT... 2.6667

C) STRUCTURE LOADING:

FX......................... 0.0000
FY......................... 0.0000
FM......................... 0.0000
FA......................... -100.0000
CONCENTRATED LOAD AT NODE... 7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.40294	0.60440
2	2.66667	3.27370	0.15000	0.22500
3	5.33333	8.18358	0.15000	0.22500
4	8.00000	12.49695	0.15000	0.22500
5	10.66666	15.68683	0.15000	0.22500
6	13.33333	18.42828	0.15000	0.22500
7	16.00000	20.90140	0.15000	0.22500
8	18.66666	23.20117	0.15000	0.22500
9	21.33333	25.28195	0.15000	0.22500
10	23.99998	27.28166	0.15000	0.22500
11	26.66666	29.15797	0.15000	0.22500
12	29.33331	30.54808	0.15000	0.22500
13	31.99998	31.99998	0.21518	0.32277

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 11.308629

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	9208.7	42801.0	52009.6
2	25084.6	3375.4	28460.0
3	25443.1	6676.3	32119.3
4	25460.7	9375.9	34836.6
5	25436.8	3831.6	29268.3
6	25386.7	18147.5	43534.2
7	25363.4	26649.6	52012.9
8	25365.1	25377.8	50742.9
9	25379.9	20239.0	45618.8
10	25414.8	1767.0	27181.8
11	25514.8	25373.4	50888.2
12	25579.6	26397.0	51976.7
13	17825.3	34401.1	52226.4

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	1
13	1	1	1

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.113937E-01	-0.117173E-01	-0.568126E-02
3	0.342988E-01	-0.295478E-01	-0.443323E-02
4	0.505141E-01	-0.446311E-01	-0.473744E-02
5	0.648279E-01	-0.611976E-01	-0.524971E-02
6	0.749570E-01	-0.755686E-01	-0.338176E-02
7	0.768717E-01	-0.821558E-01	0.238845E-03

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8	0.693485E-01	-0.779902E-01	0.431015E-02
9	0.544237E-01	-0.635129E-01	0.773890E-02
10	0.346055E-01	-0.417868E-01	0.936886E-02
11	0.157549E-01	-0.198010E-01	0.765835E-02
12	0.522320E-02	-0.514698E-02	0.419866E-02
13	0.000000E+00	0.000000E+00	0.000000E+00

FILE: FILE CASE9 A

OPTIMIZATION NO.9

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	18.475	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS... 13
NO OF DEGREES OF FREEDOM.... 39
HORIZ. LENGTH PER ELEMENT... 2.6667

C) STRUCTURE LOADING:

FX.......................... 0.0000
FY.......................... 0.0000
FM.......................... 0.0000
FA.......................... -100.0000
CONCENTRATED LOAD AT NODE... 7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15003	0.22504
2	2.66667	2.28090	0.15000	0.22500
3	5.33333	4.52538	0.15025	0.22537
4	8.00000	6.57364	0.15012	0.22518
5	10.66666	8.44217	0.15017	0.22526
6	13.33333	10.14746	0.15014	0.22521
7	16.00000	11.70146	0.15023	0.22535
8	18.66666	13.11621	0.15000	0.22500
9	21.33333	14.39744	0.15005	0.22507
10	23.99998	15.55445	0.15024	0.22535
11	26.66666	16.59473	0.15000	0.22500
12	29.33331	17.52634	0.15000	0.22500
13	31.99998	18.47499	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 8.390676

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	33701.0	0.1	33701.1
2	33704.7	18242.0	51946.6
3	33646.0	18393.0	52039.0
4	33675.5	18362.1	52037.6
5	33663.0	18380.3	52043.3
6	33671.2	18369.3	52040.5
7	33649.9	18380.3	52030.2
8	33702.3	18278.8	51981.2
9	33692.0	18333.3	52025.3
10	33649.5	18317.5	51967.0
11	33703.4	18112.9	51816.3
12	33701.9	17257.1	50959.1
13	33699.5	0.1	33699.6

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.137583E-01
2	0.273039E-01	-0.379869E-01	-0.123359E-01
3	0.488191E-01	-0.696248E-01	-0.949834E-02
4	0.624682E-01	-0.935884E-01	-0.675184E-02
5	0.696080E-01	-0.110146E+00	-0.409314E-02
6	0.713910E-01	-0.119528E+00	-0.150820E-02
7	0.687846E-01	-0.121933E+00	0.101241E-02

FILE: FILE CASE9 A

8	0.626172E-01	-0.117539E+00	0.347161E-02
9	0.536328E-01	-0.106513E+00	0.587863E-02
10	0.424687E-01	-0.889785E-01	0.824613E-02
11	0.296891E-01	-0.650603E-01	0.105634E-01
12	0.158094E-01	-0.349536E-01	0.127837E-01
13	0.000000E+00	0.000000E+00	0.138690E-01

FILE: FILE CASE10 A

OPTIMIZATION NO.10

OPTIMIZATION SOLUTION

A) PROBLEM PARAMETERS:

HORIZ. SPAN:	32.000	YOUNGS MODULUS:	30000000.0
VERT. SPAN :	55.426	YIELD STRENGTH:	52000.0
NO OF DESIGN VAR:	24	NO OF ELEMENTS:	12

B) DERIVED CONSTANTS:

NO OF SYSTEM NODAL POINTS... 13
NO OF DEGREES OF FREEDOM.... 39
HORIZ. LENGTH PER ELEMENT... 2.6667

C) STRUCTURE LOADING:

FX......................... 0.0000
FY......................... 0.0000
FM......................... 0.0000
FA......................... -100.0000
CONCENTRATED LOAD AT NODE... 7

D) ELEMENTAL DIMENSIONS AND STRESS DISTRIBUTION:

NODE	X-COORD	Y-COORD	BASE	AREA
1	0.00000	0.00000	0.15000	0.22500
2	2.66667	11.92522	0.15000	0.22500
3	5.33333	19.67612	0.15000	0.22500
4	8.00000	26.02310	0.15000	0.22500
5	10.66666	30.88034	0.15000	0.22500
6	13.33333	34.78506	0.20405	0.30607
7	16.00000	39.13298	0.15000	0.22500
8	18.66666	42.25943	0.15000	0.22500
9	21.33333	45.35509	0.15000	0.22500
10	23.99998	48.17604	0.15000	0.22500
11	26.66666	50.91873	0.15000	0.22500
12	29.33331	53.12798	0.15000	0.22500
13	31.99998	55.42598	0.15000	0.22500

E) OBJECTIVE FUNCTION:

TOTAL STRUCTURE VOLUME: 15.148624

NODE	NORMAL STRESS	BENDING STRESS	TOTAL
1	37992.2	0.1	37992.3
2	38009.5	13993.7	52003.2
3	38045.6	13552.0	51597.5
4	38084.5	9528.0	47612.5
5	38055.6	1160.7	39216.3
6	27936.6	24086.0	52022.6
7	38030.6	13224.5	51255.2
8	38061.4	13822.2	51883.6
9	38059.8	13996.9	52056.7
10	38064.1	13974.1	52038.1
11	38067.4	4071.1	42138.4
12	38068.1	13946.2	52014.3
13	38069.7	0.1	38069.9

F) BOUNDARY CONDITIONS:

NODE	X-DISPL	Y-DISPL	SLOPE
1	1	1	0
13	1	1	0

G) SOLUTION VECTOR:

NODE	X-DISPL	Y-DISPL	SLOPE
1	0.000000E+00	0.000000E+00	-0.101930E-01
2	0.103071E+00	-0.389057E-01	-0.639297E-02
3	0.129693E+00	-0.590526E-01	-0.137548E-02
4	0.129351E+00	-0.683836E-01	-0.759869E-03
5	0.133686E+00	-0.787921E-01	-0.207601E-02
6	0.135401E+00	-0.861105E-01	0.737877E-03
7	0.122050E+00	-0.843440E-01	0.261456E-02

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8	0.111694E+00	-0.823635E-01	0.266911E-02
9	0.961471E-01	-0.758125E-01	0.519496E-02
10	0.747049E-01	-0.623201E-01	0.760786E-02
11	0.485993E-01	-0.437086E-01	0.844971E-02
12	0.262193E-01	-0.235824E-01	0.920963E-02
13	0.000000E+00	0.000000E+00	0.103006E-01

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